Chapter 11: Probability and Statistics

Section 11.2-11.3: Probability

OBJ: Calculate the probability of an event.

### Probability Basics

**Definition:** The likelihood that an event will occur.

**Notation:** \( P(A) \) = probability that event A will occur

\[
\begin{align*}
0 & \leq P(A) \leq 1 \\
0\% & \leq 50\% \leq 100\%
\end{align*}
\]

<table>
<thead>
<tr>
<th>( P(A) )</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Event will never occur</td>
</tr>
<tr>
<td>0.5</td>
<td>Event is as equally likely to occur as to not occur</td>
</tr>
<tr>
<td>1.0</td>
<td>Event will absolutely occur</td>
</tr>
</tbody>
</table>

### Complement

**Definition:** Probability of an event NOT occurring.

**Notation:** \( A' \) ("Complement of A" or "Not A")

\[
P(A') = 1 - P(A)
\]

### Calculating Probability

\[
P(A) = \frac{\text{# of ways event A can occur (favorable outcomes)}}{\text{# of total outcomes possible}}
\]

**Types of probability:**
- **Theoretical:** likelihood of an event happening based on all possible outcomes.
- **Experimental:** data from observations.
- **Geometric:** area or volume.

### Geometric Probability

\[
P(\text{shaded region}) = \frac{\text{Shaded Area}}{\text{Total Area}}
\]

**Area of a...**
- **Square:** \( A = l \times w \)
- **Triangle:** \( A = \frac{1}{2} b \times h \)
- **Circle:** \( A = \pi r^2 \)
### Theoretical vs. Experimental

<table>
<thead>
<tr>
<th>Theoretical Probability of Rolling a Single 6-Sided Die</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Outcomes</td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Experimental Probability

With your shoulder buddy...

1. Roll the die 10 times (no more, no less!).
2. Record the data on the sheet (line 3).
3. Calculate your experimental probability (line 4).
   - Total outcomes should be 10.

### Example *(not included in closed notes):*

A jar contains 5 red marbles, 3 green marbles, 2 yellow marbles and 1 blue marble. Find the probability of randomly drawing the given type of marble.

- a) \( P(\text{Yellow}) = \frac{2}{11} \)
- b) \( P(\text{Green or Yellow}) = \frac{5}{11} \)

### Example *(not included in closed notes):*

A card is drawn randomly from a standard 52-card deck. Find the probability of drawing the given card.

- a) \( P(\text{Queen of Diamonds}) = \frac{1}{52} \)
- b) \( P(\text{A card that is not a five}) = \frac{48}{52} \text{ or } 1 - \frac{4}{52} \)

### Example *(not included in closed notes):*

You tossed a coin 10 times and recorded a head 3 times, a tail 7 times.

- a) \( P(\text{Head}) = \frac{3}{10} \)
- b) \( P(\text{Tail}) = \frac{7}{10} \)

### Example 1: *(See closed notes.)*

Calculate the probability of rolling 2 dice and getting a sum of 5 or 12?

- There are 36 possible outcomes.
- Sample Space: \( \{11, 12, 13, 14, 15, 16, 21, 22, 23, 24, 25, 26, 31, 32, 33, 34, 35, 36, 41, 42, 43, 44, 45, 46, 51, 52, 53, 54, 55, 56, 61, 62, 63, 64, 65, 66\} \)
- Favorable outcomes: \( \{11, 12, 15, 16, 24, 33, 42, 51, 56, 65\} \)
- Total number of favorable outcomes: 10
- Probability: \( \frac{10}{36} = \frac{5}{18} \)
Example 2: (See closed notes.)

Five cards are drawn from a standard 52-card deck.

a) What is the probability that all of the cards are diamonds?

\[
\frac{\binom{13}{5} \cdot \binom{39}{0}}{\binom{52}{5}} = \frac{1287 \cdot 1}{2,598,960} = 0.05% 
\]

b) What is the probability that the two of the cards are red and three of the cards are black?

\[
\frac{\binom{26}{2} \cdot \binom{26}{3}}{\binom{52}{5}} = \frac{525 \cdot 6,578}{2,598,960} = 3.05\%
\]

Example 3: (See closed notes.)

There are 9 students on the math team. Five of the students are seniors and four are juniors. What is the probability that in a group of five students chosen for the competition, four of them are seniors?

\[
\frac{\binom{5}{4} \cdot \binom{4}{1}}{\binom{9}{5}} = \frac{5 \cdot 4}{252} = 0.1587% 
\]

Example 4: (See closed notes.)

Find the probability that a randomly thrown dart would hit the shaded region of the target shown below.

\[
\frac{\pi \cdot 2^2}{\pi \cdot 8^2} = \frac{4}{32} = \frac{1}{8} = 12.5%
\]

Example 5: (See closed notes.)

Find the probability of randomly selecting one card from a standard 52-card deck and the card is a face card or a red card.

\[
P(\text{face or red}) = \frac{12}{52} + \frac{26}{52} - \frac{6}{52} = \frac{32}{52} = \frac{8}{13}
\]

Example 6: (See closed notes.)

Find the probability of drawing a heart, then a red card from a standard 52-card deck.

(a) with replacement

\[
P(\text{heart} \text{ then red}) = \frac{13}{52} \cdot \frac{26}{52} = \frac{13}{52} \cdot \frac{26}{52} = \frac{13}{52} \cdot \frac{26}{52}
\]

(b) without replacement

\[
P(\text{heart} \text{ then red}) = \frac{13}{52} \cdot \frac{26}{51} = \frac{13}{52} \cdot \frac{26}{51}
\]
Example 7: (See closed notes.)
In a survey of 200 pet owners, 103 owned dogs, 88 owned cats, 25 owned birds, and 18 owned reptiles.

Of the respondents, 119 owned a dog or a reptile. What is the probability that they owned a dog and a reptile?

Coursework:

- Probability Mini-Quiz #2 beginning of next class!