1.3 Linear Equations in Two Variables

Objective: SWBAT use slope to graph linear equations in two variables, find the slope of a line given two points, use slope intercept form and point-slope form.

Vocabulary:

Rate of Change: shows the relationship between two changing quantities.

Rate of change = \( \frac{\text{Change in dependent variable}}{\text{Change in independent variable}} \)

Independent variable: x-value, domain, does not depend on variation of another value
Dependent variable: y-value, range, depends on variation of independent variable

Slope: rate of change

Slope = \( \frac{\text{Vertical change}}{\text{Horizontal change}} = \frac{\text{Rise}}{\text{Run}} \)

Slope:
Calculating slope given two points.

Given 2 points \((x_1, y_1)\) and \((x_2, y_2)\)

The formula for slope \(m\) is:

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

Examples 1 and 2:
Calculate the slope.

\((-2, 6) \rightarrow (3, 5)\)

\[
\frac{5-6}{3-(-2)} = \frac{-1}{5}
\]

\((3, 4) \rightarrow (3, 1)\)

\[
\frac{4-4}{3-3} = \frac{0}{0}
\]

\((-1, 2) \rightarrow (2, 2)\)

\[
\frac{2-2}{2-(-1)} = \frac{0}{3}
\]

Slope:
Undefined

Vertical line

Zero slope

Horizontal line

Warm-Up:
If \(f(x) = (3x + 7)^2\), then \(f(1) = ?\)

A. 10  B. 16  C. 58  D. 79  E. 100

The monthly fees for single rooms at 5 colleges are $370, $310, $380, $340, and $310, respectively. What is the mean of these monthly fees?

F. $310  G. $340  H. $342  J. $350  K. $380
Application:  
Example 10  
You are driving on a road that has a 6% uphill grade. This means that the slope of the road is 6/100. Approximate the amount of vertical change in your position if you drive 200 feet.

$$\text{Vertical change} = \frac{6}{100} \times 200 = 12$$

My little brother’s roofing crew wears a harness if the pitch is steeper than 1 by 1. Explain this in terms of slope.

How many triangles do you see?  
By adding just two lines you can get 10 triangles.  
Where should you draw the 2 lines?

Slope-Intercept Form:

$$y = mx + b$$

Slope  
$y$ - intercept

Examples 5 & 6:

Find the slope and $y$-intercept of the graph of each equation.

$y = 3x + 1$  
$y = 6x + 0$

Examples 7 and 8:

Identify the slope and $y$-intercept and use them to graph the line.

$y = -5x + 4$  
$y = \frac{2x - 6}{3}$
Write an equation in slope-intercept form of the line with given slope \( m \) and y-intercept \( b \).

\( m = 0, b = -5 \) \quad \begin{align*} y &= 0x - 5 \quad \text{(5, 2) } m \text{ is undefined} \\ y &= -5 \end{align*} \]

Vocabulary:

**Point-Slope Form:** an equation of a line without using the y-intercept. Given a point on the line and slope

\[
y - y_1 = m(x - x_1)
\]

**Example 12:**
Write the equation of the line with a slope of 2/3 passing through (8, -4)

\[
y + 4 = \frac{2}{3}(x - 8) \Rightarrow \begin{array}{c} y + 4 = \frac{2}{3}x - \frac{16}{3} \\ -4 = -4 \longrightarrow \end{array} \frac{2}{3}x - \frac{16}{3} \]

Parallel/Perpendicular Lines:

Two distinct non-vertical lines are **parallel** if and only if their slopes are equal.

Two non-vertical lines are **perpendicular** if and only if their slopes are negative reciprocals of each other.

13. Write the equation of the line passing through the points (5, -1), (-5, 5)

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - (-1)}{-5 - 5} = \frac{6}{-10} = -\frac{3}{5}
\]

\[
y - 5 = -\frac{3}{5}(x + 5) \Rightarrow \begin{array}{c} y - 5 = -\frac{3}{5}x - \frac{3}{5} \\ -\frac{3}{5} \quad \frac{9}{5} \end{array} \]

14. Write the equation of the line with a slope of -1/2 passing through the point (2, 3).

\[
y - 3 = -\frac{1}{2}(x - 2) \Rightarrow \begin{array}{c} y - 3 = -\frac{1}{2}x + 1 \\ +3 \end{array} \quad \begin{array}{c} y = -\frac{1}{2}x + 4 \end{array}
\]

15. Write the equation of the line parallel to \( 2y - 3y = 5 \) passing through (2, -1)

\[
y + 1 = \frac{2}{3}(x - 2) \Rightarrow \begin{array}{c} y + 1 = \frac{2}{3}x - \frac{4}{3} \\ +1 \end{array} \quad \begin{array}{c} y = \frac{2}{3}x - \frac{2}{3} \end{array}
\]

16. Determine if the lines are parallel, perpendicular or neither.

L1: (0, -1), (5, 9) and L2: (0, 3) and (4, 1)

\[
\begin{array}{c}
\frac{q_1 - 1}{s - 0} = \frac{10}{5} = \frac{2}{1} \quad \frac{1 - 3}{q_2 - 0} = \frac{-2}{-1} = 2
\end{array}
\]
Coursework:

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