1.4  EXERCISES

VOCABULARY: Fill in the blanks.

1. A relation that assigns to each element \( x \) from a set of inputs, or ________, exactly one element \( y \) in a set of outputs, or ________, is called a ________. domain; range; function

2. Functions are commonly represented in four different ways, ________, ________, ________, and ________. See margin.

3. For an equation that represents \( y \) as a function of \( x \), the set of all values taken on by the ________ variable \( x \) is the domain, and the set of all values taken on by the ________ variable \( y \) is the range. independent; dependent

4. The function given by
   \[
   f(x) = \begin{cases} 
   2x - 1, & x < 0 \\
   x^2 + 4, & x \geq 0 
   \end{cases}
   \]

is an example of a ________ function. piecewise-defined

5. If the domain of the function \( f \) is not given, then the set of values of the independent variable for which the expression is defined is called the ________. implied domain

6. In calculus, one of the basic definitions is that of a ________, given by \( \frac{f(x + h) - f(x)}{h} \), \( h \neq 0 \). difference quotient

SKILLS AND APPLICATIONS

In Exercises 7–10, is the relationship a function?

7. Domain Range
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 5 \quad \text{Yes} \\
   -1 & 6 \\
   0 & 7 \\
   1 & 8 \\
   
   \end{array}
   \]

8. Domain Range
   \[
   \begin{array}{c|c}
   x & y \\
   \hline
   -2 & 3 \quad \text{No} \\
   -1 & 4 \\
   0 & 5 \\
   
   \end{array}
   \]

9. Domain Range
   National League
   - Cubs
   - Pirates
   - Dodgers
   American League
   - Orioles
   - Yankees
   - Twins

10. Domain Range
    (Year)
    - 1999: 10
    - 2000: 12
    - 2001: 15
    - 2002: 16
    - 2003: 21
    - 2004: 24
    - 2005: 27
    - 2006: 10
    - 2007: 3
    - 2008: 2

In Exercises 11–14, determine whether the relation represents \( y \) as a function of \( x \).

11. Input, \( x \) | Output, \( y \)
    \[
    \begin{array}{c|c|c|c|c|c}
    x & -2 & -1 & 0 & 1 & 2 \\
    \hline
    y & -8 & -1 & 0 & 1 & 8 \\
    \end{array}
    \]

   Yes, each input value has exactly one output value.

12–14. See margin.

12. Input, \( x \) | Output, \( y \)
    \[
    \begin{array}{c|c|c|c|c|c}
    x & 0 & 1 & 2 & 1 & 0 \\
    \hline
    y & -4 & -2 & 0 & 2 & 4 \\
    \end{array}
    \]

13. Input, \( x \) | Output, \( y \)
    \[
    \begin{array}{c|c|c|c|c|c}
    x & 10 & 7 & 4 & 7 & 10 \\
    \hline
    y & 3 & 6 & 9 & 12 & 15 \\
    \end{array}
    \]

14. Input, \( x \) | Output, \( y \)
    \[
    \begin{array}{c|c|c|c|c|c}
    x & 0 & 3 & 9 & 12 & 15 \\
    \hline
    y & 3 & 3 & 3 & 3 & 3 \\
    \end{array}
    \]

In Exercises 15 and 16, which sets of ordered pairs represent functions from \( A \) to \( B \)? Explain. 15–16. See margin.

15. \( A = \{0, 1, 2, 3\} \) and \( B = \{-2, -1, 0, 1, 2\} \)
    (a) \{\{0, 1\}, \{1, -2\}, \{2, 0\}, \{3, 2\}\}
    (b) \{\{0, -1\}, \{2, 2\}, \{1, -2\}, \{3, 0\}, \{1, 1\}\}
    (c) \{\{0, 0\}, \{1, 0\}, \{2, 0\}, \{3, 0\}\}
    (d) \{\{0, 2\}, \{3, 0\}, \{1, 1\}\}

16. \( A = \{a, b, c\} \) and \( B = \{0, 1, 2, 3\} \)
    (a) \{\{(a, 1\}, \{(c, 2\}, \{(c, 3\}, \{(b, 3\}\}
    (b) \{\{(a, 1\}, \{(b, 2\}, \{(c, 3\}\}
    (c) \{\{(1, a\}, \{(0, a\}, \{(2, c\}, \{(3, b\}\}
    (d) \{\{(c, 0\}, \{(b, 0\}, \{(a, 3\}\}
    (e) \{\{(a, 0\}, \{(b, 0\}, \{(a, 0\}\}

CIRCULATION OF NEWSPAPERS In Exercises 17 and 18, use the graph, which shows the circulation (in millions) of daily newspapers in the United States. (Source: Editor & Publisher Company)

![Graph showing circulation of newspapers](image)

17. Is the circulation of morning newspapers a function of the year? Is the circulation of evening newspapers a function of the year? Explain. **See margin.**

18. Let \( f(x) \) represent the circulation of evening newspapers in year \( x \). Find \( f(2002) \). **9 million**

In Exercises 19–36, determine whether the equation represents \( y \) as a function of \( x \).

19. \( x^2 + y^2 = 4 \) **Not a function**

20. \( x^2 - y^2 = 16 \) **Not a function**

21. \( x^2 + y = 4 \) **Function**

22. \( y - 4x^2 = 36 \) **Function**

23. \( 2x + 3y = 4 \) **Function**

24. \( 2x + 5y = 10 \) **Function**

25. \((x + 2)^2 + (y - 1)^2 = 25 \) **Not a function**

26. \((x - 2)^2 + y^2 = 4 \) **Not a function**

27. \( y^2 = x^2 - 1 \) **Not a function**

28. \( x + y^2 = 4 \) **Not a function**

29. \( y = \sqrt{16 - x^2} \) **Function**

30. \( y = \sqrt{x + 5} \) **Function**

31. \( y = |4 - x| \) **Function**

32. \( |y| = 4 - x \) **Not a function**

33. \( x = 14 \) **Not a function**

34. \( y = -75 \) **Function**

35. \( y + 5 = 0 \) **Function**

36. \( x - 1 = 0 \) **Not a function**

In Exercises 37–52, evaluate the function at each specified value of the independent variable and simplify.

37. \( f(x) = 2x - 3 \)
   - (a) \( f(1) = 1 \)
   - (b) \( f(-3) = -9 \)
   - (c) \( f(x - 1) = 2x - 5 \)

38. \( g(y) = 7 - 3y \)
   - (a) \( g(0) = 7 \)
   - (b) \( g(\frac{7}{3}) = 0 \)
   - (c) \( g(s + 2) = 1 - 3s \)

39. \( V(r) = \frac{4}{3}\pi r^3 \)
   - (a) \( V(3) = 36\pi \)
   - (b) \( V(\frac{3}{2}) = \frac{9}{2}\pi \)
   - (c) \( V(2r) = \frac{32}{3}\pi r^3 \)

40. \( S(r) = 4\pi r^2 \)
   - (a) \( S(2) = 16\pi \)
   - (b) \( S(\frac{1}{2}) = \pi \)
   - (c) \( S(3r) = 36\pi r^2 \)

41. \( g(t) = 4t^2 - 3t + 5 \)
   - (a) \( g(2) = 15 \)
   - (b) \( g(t - 2) = 4t^2 - 19t + 27 \)
   - (c) \( g(t) - g(2) = 4t^2 - 3t - 10 \)

42. \( h(t) = t^2 - 2t \)
   - (a) \( h(2) = 0 \)
   - (b) \( h(1.5) = 0.75 \)
   - (c) \( h(x + 2) \)

43. \( f(y) = 3 - \sqrt{y} \)
   - (a) \( f(4) = 1 \)
   - (b) \( f(0.25) = 2.5 \)
   - (c) \( f(4x^3 - 2|x|) \)

44. \( f(x) = \sqrt{x + 8} + 2 \)
   - (a) \( f(-8) = 2 \)
   - (b) \( f(1) = 5 \)
   - (c) \( f(x - 8) = \sqrt{x + 2} \)

45. \( q(x) = \frac{1}{(x^2 - 9)} \) **See margin.**
   - (a) \( q(0) \)
   - (b) \( q(3) \)
   - (c) \( q(y + 3) \)

46. \( q(t) = \frac{(2t^2 + 3)}{t^2} \) **See margin.**
   - (a) \( q(2) \)
   - (b) \( q(0) \)
   - (c) \( q(-x) \)

47. \( f(x) = |x|/x \) **See margin.**
   - (a) \( f(2) \)
   - (b) \( f(-2) \)
   - (c) \( f(x) = x - 1 \)

48. \( f(x) = |x| + 4 \)
   - (a) \( f(2) = 6 \)
   - (b) \( f(-2) = 6 \)
   - (c) \( f(x^2) = x^2 + 4 \)

49. \( f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \geq 0 \end{cases} \)
   - (a) \( f(-1) = -1 \)
   - (b) \( f(0) = 2 \)
   - (c) \( f(2) = 6 \)

50. \( f(x) = \begin{cases} x^2 + 2, & x \leq 1 \\ 2x^2 + 2, & x > 1 \end{cases} \)
   - (a) \( f(-2) = 6 \)
   - (b) \( f(1) = 3 \)
   - (c) \( f(2) = 10 \)

51. \( f(x) = \begin{cases} 3x - 1, & x < -1 \\ x^2, & x \geq 1 \end{cases} \)
   - (a) \( f(-2) = 7 \)
   - (b) \( f(-\frac{1}{2}) = 4 \)
   - (c) \( f(3) = 9 \)

52. \( f(x) = \begin{cases} 4 - 5x, & x \leq -2 \\ x^2 + 1, & x \geq 2 \end{cases} \)
   - (a) \( f(-3) = 19 \)
   - (b) \( f(4) = 17 \)
   - (c) \( f(-1) = 0 \)

In Exercises 53–58, complete the table.

53. \( f(x) = x^2 - 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(-2)</th>
<th>(-1)</th>
<th>(0)</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>4</td>
<td>2</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

54. \( g(x) = \sqrt{x - 3} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g(x) )</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

55. \( h(t) = \frac{1}{2}|t + 3| \)

<table>
<thead>
<tr>
<th>( t )</th>
<th>(-5)</th>
<th>(-4)</th>
<th>(-3)</th>
<th>(-2)</th>
<th>(-1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( h(t) )</td>
<td>1</td>
<td>(\frac{1}{2})</td>
<td>0</td>
<td>(\frac{1}{2})</td>
<td>1</td>
</tr>
</tbody>
</table>
56. \( f(s) = \frac{|s - 2|}{s - 2} \)

<table>
<thead>
<tr>
<th>( s )</th>
<th>0</th>
<th>1</th>
<th>( \frac{3}{2} )</th>
<th>( \frac{5}{2} )</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(s) )</td>
<td>-1</td>
<td>-1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

57. \( f(x) = \begin{cases} \frac{3x}{2} + 4, & x \leq 0 \\ (x - 2)^2, & x > 0 \end{cases} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>5</td>
<td>( \frac{9}{2} )</td>
<td>4</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

58. \( f(x) = \begin{cases} 9 - x^2, & x < 3 \\ x - 3, & x \geq 3 \end{cases} \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>8</td>
<td>5</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

In Exercises 59–66, find all real values of \( x \) such that \( f(x) = 0 \).

59. \( f(x) = 15 - 3x \)

60. \( f(x) = 5x + 1 \)

61. \( f(x) = \frac{3x - 4}{5} \)

62. \( f(x) = \frac{12 - x^2}{5} \)

63. \( f(x) = x^2 - 9 \)

64. \( f(x) = x^2 - 8x + 15 \)

65. \( f(x) = x^3 - x \)

66. \( f(x) = x^3 - x^2 - 4x + 4 \)

In Exercises 67–70, find the value(s) of \( x \) for which \( f(x) = g(x) \).

67. \( f(x) = x^2, \quad g(x) = x + 2 \)

68. \( f(x) = x^2 + 2x + 1, \quad g(x) = 7x - 5 \)

69. \( f(x) = x^4 - 2x^2, \quad g(x) = 2x^2 \)

70. \( f(x) = \sqrt{x} - 4, \quad g(x) = 2 - x \)

In Exercises 71–82, find the domain of the function.

71. \( f(x) = 5x^2 + 2x - 1 \)

72. \( g(x) = 1 - 2x^2 \)

73. \( h(t) = \frac{4}{t} \)

74. \( s(y) = \frac{3y}{y + 5} \)

75. \( g(y) = \sqrt{y} - 10 \)

76. \( f(t) = \frac{3}{t} + 4 \)

77. \( g(x) = \frac{1}{x - x^2} \)

78. \( h(x) = \frac{10}{x^2 - 2x} \)

79. \( f(s) = \frac{s - 1}{s - 4} \)

80. \( f(x) = \frac{x + 6}{6 + x} \)

81. \( f(x) = \frac{x - 4}{\sqrt{x}} \)

82. \( f(x) = \frac{x + 2}{\sqrt{x} - 10} \)

71–82. See margin.

In Exercises 83–86, assume that the domain of \( f \) is the set \( A = \{-2, -1, 0, 1, 2\} \). Determine the set of ordered pairs that represents the function \( f \). See margin.

83. \( f(x) = x^2 \)

84. \( f(x) = (x - 3)^2 \)

85. \( f(x) = |x| + 2 \)

86. \( f(x) = |x + 1| \)

87. **GEOMETRY** Write the area \( A \) of a square as a function of its perimeter \( P \). See margin.

88. **GEOMETRY** Write the area \( A \) of a circle as a function of its circumference \( C \). See margin.

89. **MAXIMUM VOLUME** An open box of maximum volume is to be made from a square piece of material 24 centimeters on a side by cutting equal squares from the corners and turning up the sides (see figure). See margin.

(a) The table shows the volumes \( V \) (in cubic centimeters) of the box for various heights \( x \) (in centimeters). Use the table to estimate the maximum volume.

<table>
<thead>
<tr>
<th>Height, ( x )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volume, ( V )</td>
<td>484</td>
<td>800</td>
<td>972</td>
<td>1024</td>
<td>980</td>
<td>864</td>
</tr>
</tbody>
</table>

(b) Plot the points \((x, V)\) from the table in part (a). Does the relation defined by the ordered pairs represent \( V \) as a function of \( x \)?

(c) If \( V \) is a function of \( x \), write the function and determine its domain.

90. **MAXIMUM PROFIT** The cost per unit in the production of an MP3 player is $60. The manufacturer charges $90 per unit for orders of 100 or less. To encourage large orders, the manufacturer reduces the charge by $0.15 per MP3 player for each unit ordered in excess of 100 (for example, there would be a charge of $87 per MP3 player for an order size of 120).

(a) The table shows the profits \( P \) (in dollars) for various numbers of units ordered, \( x \). Use the table to estimate the maximum profit. See margin.

<table>
<thead>
<tr>
<th>Units, ( x )</th>
<th>110</th>
<th>120</th>
<th>130</th>
<th>140</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit, ( P )</td>
<td>3135</td>
<td>3240</td>
<td>3315</td>
<td>3360</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Units, ( x )</th>
<th>150</th>
<th>160</th>
<th>170</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit, ( P )</td>
<td>3375</td>
<td>3360</td>
<td>3315</td>
</tr>
</tbody>
</table>