2.1 EXERCISES

**VOCABULARY:** Fill in the blanks.

1. Linear, constant, and squaring functions are examples of _______ functions.
2. A polynomial function of degree \( n \) and leading coefficient \( a_n \) is a function of the form 
   \[ f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0 \] 
   where \( n \) is a _______ number and \( a_n, a_{n-1}, \ldots, a_1, a_0 \) are _______ numbers.
3. A quadratic function is a second-degree polynomial function, and its graph is called a _______.
4. The graph of a quadratic function is symmetric about its _______.
5. If the graph of a quadratic function opens upward, then its leading coefficient is _______ and the vertex of the graph is a _______.
6. If the graph of a quadratic function opens downward, then its leading coefficient is _______ and the vertex of the graph is a _______.

**SKILLS AND APPLICATIONS**

In Exercises 7–12, match the quadratic function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

- 7. \( f(x) = (x - 2)^2 \) **e**
- 8. \( f(x) = (x + 4)^2 \) **c**
- 9. \( f(x) = x^2 - 2 \) **b**
- 10. \( f(x) = (x + 1)^2 - 2 \) **a**
- 11. \( f(x) = 4 - (x - 2)^2 \) **f**
- 12. \( f(x) = -(x - 4)^2 \) **d**

In Exercises 17–34, sketch the graph of the quadratic function without using a graphing utility. Identify the vertex, axis of symmetry, and x-intercept(s). 17–34. See margin.

- 17. \( f(x) = 1 - x^2 \)
- 18. \( g(x) = x^2 - 8 \)
- 19. \( f(x) = x^2 + 7 \)
- 20. \( h(x) = 12 - x^2 \)
- 21. \( f(x) = \frac{1}{4}x^2 - 4 \)
- 22. \( f(x) = 16 - \frac{1}{4}x^2 \)
- 23. \( f(x) = (x + 4)^2 - 3 \)
- 24. \( f(x) = (x - 6)^2 + 8 \)
- 25. \( h(x) = x^2 - 8x + 16 \)
- 26. \( g(x) = x^2 + 2x + 1 \)
- 27. \( f(x) = x^2 - x + \frac{5}{4} \)
- 28. \( f(x) = x^2 + 3x + \frac{1}{4} \)
- 29. \( f(x) = -x^2 + 2x + 5 \)
- 30. \( f(x) = -x^2 - 4x + 1 \)
- 31. \( h(x) = 4x^2 - 4x + 21 \)
- 32. \( f(x) = 2x^2 - x + 1 \)
- 33. \( f(x) = \frac{1}{4}x^2 - 2x - 12 \)
- 34. \( f(x) = -\frac{1}{3}x^2 + 3x - 6 \)

In Exercises 35–42, use a graphing utility to graph the quadratic function. Identify the vertex, axis of symmetry, and x-intercepts. Then check your results algebraically by writing the quadratic function in standard form. 35–42. See margin.

- 35. \( f(x) = -(x^2 + 2x - 3) \)
- 36. \( f(x) = -(x^2 + x - 30) \)
- 37. \( g(x) = x^2 + 8x + 11 \)
- 38. \( f(x) = x^2 + 10x + 14 \)
- 39. \( f(x) = 2x^2 - 16x + 31 \)
- 40. \( f(x) = -4x^2 + 24x - 41 \)
- 41. \( g(x) = \frac{1}{2}x^2 + 4x - 2 \)
- 42. \( f(x) = \frac{3}{5}(x^2 + 6x - 5) \)
In Exercises 43–46, write an equation for the parabola in standard form.

43. \[ y = -(x + 1)^2 + 4 \]
44. \[ y = (x + 2)^2 - 1 \]
45. \[ y = -2(x + 2)^2 + 2 \]
46. \[ y = 2(x - 2)^2 \]

In Exercises 47–56, write the standard form of the equation of the parabola that has the indicated vertex and whose graph passes through the given point.

47. Vertex: \((-2, 5)\); point: \((0, 9)\) \[ f(x) = (x + 2)^2 + 5 \]
48. Vertex: \((4, -1)\); point: \((2, 3)\) \[ f(x) = (x - 4)^2 - 1 \]
49. Vertex: \((1, -2)\); point: \((-1, 14)\) \[ f(x) = 4(x - 1)^2 - 2 \]
50. Vertex: \((2, 3)\); point: \((0, 2)\) \[ f(x) = -\frac{1}{4}(x - 2)^2 + 3 \]
51. Vertex: \((5, 12)\); point: \((7, 15)\) \[ f(x) = \frac{5}{4}(x - 5)^2 + 12 \]
52. Vertex: \((-2, -2)\); point: \((-1, 0)\) \[ f(x) = 2(x + 2)^2 - 2 \]
53. Vertex: \((-\frac{1}{4}, \frac{3}{2})\); point: \((-2, 0)\) \[ f(x) = -\frac{5}{8}(x + \frac{1}{2})^2 + \frac{3}{2} \]
54. Vertex: \((\frac{5}{6}, -2)\); point: \((-2, 4)\) \[ f(x) = \frac{10}{81}(x - 2)^2 - \frac{3}{4} \]
55. Vertex: \((-\frac{3}{8}, 0)\); point: \((-\frac{7}{5}, -\frac{16}{5})\) \[ f(x) = -\frac{16}{3}(x + \frac{2}{5})^2 \]
56. Vertex: \((6, 6)\); point: \((\frac{61}{10}, \frac{3}{5})\) \[ f(x) = -450(x - 6)^2 + 6 \]

GRAPHICAL REASONING In Exercises 57 and 58, determine the x-intercept(s) of the graph visually. Then find the x-intercept(s) algebraically to confirm your results.

57. \[ y = x^2 - 4x - 5 \]
58. \[ y = 2x^2 + 5x - 3 \]

In Exercises 59–64, use a graphing utility to graph the quadratic function. Find the x-intercepts of the graph and compare them with the solutions of the corresponding quadratic equation when \( f(x) = 0 \). See margin.

59. \[ f(x) = x^2 - 4x \]
60. \[ f(x) = -2x^2 + 10x \]
61. \[ f(x) = x^2 - 9x + 18 \]
62. \[ f(x) = x^2 - 8x - 20 \]
63. \[ f(x) = 2x^2 - 7x - 30 \]
64. \[ f(x) = \frac{7}{10}(x^2 + 12x - 45) \]

In Exercises 65–70, find two quadratic functions, one that opens upward and one that opens downward, whose graphs have the given x-intercepts. (There are many correct answers.)

65. \((-1, 0), (3, 0)\)
66. \((-5, 0), (5, 0)\)
67. \((0, 0), (10, 0)\)
68. \((4, 0), (8, 0)\)
69. \((-3, 0), \left(\frac{3}{2}, 0\right)\)
70. \((-\frac{3}{2}, 0), (2, 0)\)

In Exercises 71–74, find two positive real numbers whose product is a maximum.

71. The sum is 110.
72. The sum is 55.
73. The sum is the square of the second is 24.
74. The sum of the first and three times the second is 42.

75. PATH OF A DIVER The path of a diver is given by

\[ y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12 \]

where \( y \) is the height (in feet) and \( x \) is the horizontal distance from the end of the diving board (in feet). What is the maximum height of the diver? 16 ft

76. HEIGHT OF A BALL The height \( y \) (in feet) of a punt football is given by

\[ y = -\frac{16}{2025}x^2 + \frac{9}{5}x + 1.5 \]

where \( x \) is the horizontal distance (in feet) from the point at which the ball is punted.

(a) How high is the ball when it is punted? 1.5 ft
(b) What is the maximum height of the punt? See margin.
(c) How long is the punt? About 228.64 ft

77. MINIMUM COST A manufacturer of lighting fixtures has daily production costs of \( C = 800 - 10x + 0.25x^2 \), where \( C \) is the total cost (in dollars) and \( x \) is the number of units produced. How many fixtures should be produced each day to yield a minimum cost? See margin.

78. MAXIMUM PROFIT The profit \( P \) (in hundreds of dollars) that a company makes depends on the amount \( x \) (in hundreds of dollars) the company spends on advertising according to the model

\[ P = 230 + 20x - 0.5x^2 \]

What expenditure for advertising will yield a maximum profit? $2000
79. MAXIMUM REVENUE The total revenue \( R \) earned (in thousands of dollars) from manufacturing handheld video games is given by

\[
R(p) = -25p^2 + 1200p
\]

where \( p \) is the price per unit (in dollars).

(a) Find the revenues when the price per unit is \( \$20, \ $25, \ $30 \), \( \$14,000,000; \ $14,375,000; \ $13,500,000\)

(b) Find the unit price that will yield a maximum revenue. What is the maximum revenue? Explain your results. \( \$24; \ $14,400,000; \ ) Answers will vary.

80. MAXIMUM REVENUE The total revenue \( R \) earned per day (in dollars) from a pet-sitting service is given by \( R(p) = -12p^2 + 150p \), where \( p \) is the price charged per pet (in dollars). See margin.

(a) Find the revenues when the price per pet is \( \$4, \ $6, \ $8 \).

(b) Find the price that will yield a maximum revenue. What is the maximum revenue? Explain your results.

81. NUMERICAL, GRAPHICAL, AND ANALYTICAL ANALYSIS A rancher has 200 feet of fencing to enclose two adjacent rectangular corrals (see figure). See margin.

(a) Write the area \( A \) of the corrals as a function of \( x \).

(b) Create a table showing possible values of \( x \) and the corresponding areas of the corral. Use the table to estimate the dimensions that will produce the maximum enclosed area.

(c) Use a graphing utility to graph the area function. Use the graph to approximate the dimensions that will produce the maximum enclosed area.

(d) Write the area function in standard form to find analytically the dimensions that will produce the maximum area.

(e) Compare your results from parts (b), (c), and (d).

82. GEOMETRY An indoor physical fitness room consists of a rectangular region with a semicircle on each end. The perimeter of the room is to be a 200-meter single-lane running track. See margin.

(a) Draw a diagram that illustrates the problem. Let \( x \) and \( y \) represent the length and width of the rectangular region, respectively.

(b) Determine the radius of each semicircular end of the room. Determine the distance, in terms of \( x \) and \( y \), around the inside edge of each semicircular part of the track.

(c) Use the result of part (b) to write an equation, in terms of \( x \) and \( y \), for the distance traveled in one lap around the track. Solve for \( y \).

(d) Use the result of part (c) to write the area \( A \) of the rectangular region as a function of \( x \). What dimensions will produce a rectangle of maximum area?

83. MAXIMUM REVENUE A small theater has a seating capacity of 2000. When the ticket price is \( \$20 \), attendance is 1500. For each \( \$1 \) decrease in price, attendance increases by 100.

(a) Write the revenue \( R \) of the theater as a function of ticket price \( x \). See margin.

(b) What ticket price will yield a maximum revenue? What is the maximum revenue? \( \$17.50; \ $30,625 \)

84. MAXIMUM AREA A Norman window is constructed by adjoining a semicircle to the top of an ordinary rectangular window (see figure). The perimeter of the window is 16 feet. \( (a) \ A = \frac{1}{8}(64x - 4x^2 - \pi x^2) \)

(a) Write the area \( A \) of the window as a function of \( x \).

(b) What dimensions will produce a window of maximum area? \( x \approx 4.48 \text{ ft}, \ y \approx 2.24 \text{ ft} \)

85. GRAPHICAL ANALYSIS From 1950 through 2005, the per capita consumption \( C \) of cigarettes by Americans (age 18 and older) can be modeled by \( C = 3565.0 + 60.30t - 1.783t^2, \ 0 \leq t \leq 55 \), where \( t \) is the year, with \( t = 0 \) corresponding to 1950. \( \text{(Source: Tobacco Outlook Report)} \) See margin.

(a) Use a graphing utility to graph the model.

(b) Use the graph of the model to approximate the maximum average annual consumption. Beginning in 1966, all cigarette packages were required by law to carry a health warning. Do you think the warning had any effect? Explain.

(c) In 2005, the U.S. population (age 18 and over) was 296,329,000. Of those, about 59,858,458 were smokers. What was the average annual cigarette consumption per smoker in 2005? What was the average daily cigarette consumption per smoker?