2.2 EXERCISES

VOCABULARY: Fill in the blanks.

1. The graphs of all polynomial functions are continuous, which means that the graphs have no breaks, holes, or gaps.
2. The **Leading Coefficient Test** is used to determine the left-hand and right-hand behavior of the graph of a polynomial function.
3. Polynomial functions of the form \( f(x) = x^n \) are often referred to as power functions.
4. A polynomial function of degree \( n \) has at most \( n \) real zeros and at most \( n - 1 \) turning points.
5. If \( x = a \) is a zero of a polynomial function \( f \), then the following three statements are true.
   (a) \( x = a \) is a solution of the polynomial equation \( f(x) = 0 \).
   (b) \( (x - a) \) is a factor of the polynomial \( f(x) \).
   (c) \( (a, 0) \) is an intercept of the graph of \( f \).
6. If a real zero of a polynomial function is of even multiplicity, then the graph of \( f \) touches the \( x \)-axis at \( x = a \), and if it is of odd multiplicity, then the graph of \( f \) crosses the \( x \)-axis at \( x = a \).
7. A polynomial function is written in standard form if its terms are written in descending order of exponents from left to right.
8. The **Intermediate Value Theorem** states that if \( f \) is a polynomial function such that \( f(a) \neq f(b) \), then, in the interval \( [a, b] \), \( f \) takes on every value between \( f(a) \) and \( f(b) \).

SKILLS AND APPLICATIONS

In Exercises 9–16, match the polynomial function with its graph. [The graphs are labeled (a), (b), (c), (d), (e), (f), (g), and (h).]

9. \( f(x) = -2x + 3 \)  
   10. \( f(x) = x^2 - 4x \) 

11. \( f(x) = -2x^2 - 5x \)  
   12. \( f(x) = 2x^3 - 3x + 1 \) 

13. \( f(x) = -\frac{1}{2}x^4 + 3x^2 \)  
   14. \( f(x) = -\frac{1}{3}x^3 + x^2 - \frac{4}{3} \) 

15. \( f(x) = x^4 + 2x^3 \)  
   16. \( f(x) = \frac{1}{3}x^5 - 2x^3 + \frac{9}{5}x \)

In Exercises 17–20, sketch the graph of \( y = x^n \) and each transformation. 17–19. See margin.

17. \( y = x^3 \)
   (a) \( f(x) = (x - 4)^3 \)  
   (b) \( f(x) = x^3 - 4 \) 
   (c) \( f(x) = -\frac{1}{2}x^3 \)  
   (d) \( f(x) = (x - 4)^3 - 4 \)

18. \( y = x^5 \)
   (a) \( f(x) = (x + 1)^5 \)  
   (b) \( f(x) = x^5 + 1 \) 
   (c) \( f(x) = 1 - \frac{1}{2}x^5 \)  
   (d) \( f(x) = -\frac{1}{2}(x + 1)^5 \)

19. \( y = x^4 \)
   (a) \( f(x) = (x + 3)^4 \)  
   (b) \( f(x) = x^4 - 3 \) 
   (c) \( f(x) = 4 - x^4 \)  
   (d) \( f(x) = \frac{1}{3}(x - 1)^4 \) 
   (e) \( f(x) = (2x)^4 + 1 \)  
   (f) \( f(x) = \left(\frac{1}{2}x\right)^4 - 2 \)

EXPLORATION 105–107. See margin.

TRUE OR FALSE? In Exercises 105–107, determine whether the statement is true or false. Justify your answer.

105. A fifth-degree polynomial can have five turning points in its graph.

106. It is possible for a sixth-degree polynomial to have only one solution.

107. The graph of the function given by

\[ f(x) = 2 + x - x^2 + x^3 - x^4 + x^5 + x^6 - x^7 \]

rises to the left and falls to the right.

108. CAPSTONE For each graph, describe a polynomial function that could represent the graph. (Indicate the degree of the function and the sign of its leading coefficient.) See margin.

(a) 
(b) 

(c) 
(d) 

109. GRAPHICAL REASONING Sketch a graph of the function given by \( f(x) = x^3 \). Explain how the graph of each function \( g \) differs (if it does) from the graph of each function \( f \). Determine whether \( g \) is odd, even, or neither. See margin.

(a) \( g(x) = f(x) + 2 \)
(b) \( g(x) = f(x + 2) \)
(c) \( g(x) = f(-x) \)
(d) \( g(x) = -f(x) \)
(e) \( g(x) = f \left( \frac{1}{2}x \right) \)
(f) \( g(x) = \frac{1}{2} f(x) \)
(g) \( g(x) = f(x^{3/4}) \)
(h) \( g(x) = (f \circ f)(x) \)

110. THINK ABOUT IT For each function, identify the degree of the function and whether the degree of the function is even or odd. Identify the leading coefficient and whether the leading coefficient is positive or negative. Use a graphing utility to graph each function. Describe the relationship between the degree of the function and the sign of the leading coefficient of the function and the right-hand and left-hand behavior of the graph of the function. See margin.

(a) \( f(x) = x^3 - 2x^2 - x + 1 \)
(b) \( f(x) = 2x^3 + 2x^2 - 5x + 1 \)
(c) \( f(x) = -2x^3 - x^2 + 5x + 3 \)
(d) \( f(x) = -x^3 + 5x - 2 \)
(e) \( f(x) = 2x^3 + 3x - 4 \)
(f) \( f(x) = x^4 - 3x^2 + 2x - 1 \)
(g) \( f(x) = x^2 + 3x + 2 \)

111. THINK ABOUT IT Sketch the graph of each polynomial function. Then count the number of zeros of the function and the numbers of relative minima and relative maxima. Compare these numbers with the degree of the polynomial. What do you observe?

(a) \( f(x) = -x^3 + 9x \)
(b) \( f(x) = x^4 - 10x^2 + 9 \)
(c) \( f(x) = x^5 - 16x \) (a)–(c). See margin.

112. Explore the transformations of the form \( g(x) = a(x - h)^5 + k \). See margin.

(a) Use a graphing utility to graph the functions \( y_1 = -\frac{3}{2}(x - 2)^3 + 1 \) and \( y_2 = \frac{3}{2}(x + 2)^5 - 3 \). Determine whether the graphs are increasing or decreasing. Explain.

(b) Will the graph of \( g \) always be increasing or decreasing? If so, is this behavior determined by \( a, h, \) or \( k? \) Explain.

(c) Use a graphing utility to graph the function given by \( H(x) = x^3 - 3x^2 + 2x + 1 \). Use the graph and the result of part (b) to determine whether \( H \) can be written in the form \( H(x) = a(x - h)^3 + k \). Explain.
20. \( y = x^6 \) See margin.
(a) \( f(x) = -\frac{1}{8}x^6 \)
(b) \( f(x) = (x + 2)^6 - 4 \)
(c) \( f(x) = x^6 - 5 \)
(d) \( f(x) = -\frac{1}{4}x^6 + 1 \)
(e) \( f(x) = \left(\frac{1}{4}x\right)^6 - 2 \)
(f) \( f(x) = (2x)^6 - 1 \)

21–30. See margin.
In Exercises 21–30, describe the right-hand and left-hand behavior of the graph of the polynomial function.

21. \( f(x) = \frac{1}{3}x^3 + 4x \)
22. \( f(x) = 2x^2 - 3x + 1 \)
23. \( g(x) = 5 - \frac{7}{2}x - 3x^2 \)
24. \( h(x) = 1 - x^6 \)
25. \( f(x) = -2.1x^5 + 4x^3 - 2 \)
26. \( f(x) = 4x^5 - 7x + 6.5 \)
27. \( f(x) = 6 - 2x + 4x^2 - 5x^3 \)
28. \( f(x) = (3x^4 - 2x + 5)/4 \)
29. \( h(t) = -\frac{3}{2}t^2 + 3t + 6 \)
30. \( f(s) = -\frac{5}{8}(s^3 + 5s^2 - 7s + 1) \)

31–34. See margin.

GRAPhICAL ANALYSIS In Exercises 31–34, use a graphing utility to graph the functions \( f \) and \( g \) in the same viewing window. Zoom out sufficiently far to show that the right-hand and left-hand behaviors of \( f \) and \( g \) appear identical.

31. \( f(x) = 3x^3 - 9x + 1 \), \( g(x) = 3x^3 \)
32. \( f(x) = -\frac{1}{3}(x^3 - 3x + 2) \), \( g(x) = -\frac{1}{3}x^3 \)
33. \( f(x) = -(x^4 - 4x^3 + 16x) \), \( g(x) = -x^4 \)
34. \( f(x) = 3x^4 - 6x^2 \), \( g(x) = 3x^4 \)

In Exercises 35–50, (a) find all the real zeros of the polynomial function, (b) determine the multiplicity of each zero and the number of turning points of the graph of the function, and (c) use a graphing utility to graph the function and verify your answers. 35–50. See margin.

35. \( f(x) = x^2 - 36 \)
36. \( f(x) = 81 - x^2 \)
37. \( h(t) = t^2 - 6t + 9 \)
38. \( f(x) = x^2 + 10x + 25 \)
39. \( f(x) = \frac{1}{3}x^2 + \frac{1}{3}x - \frac{3}{2} \)
40. \( f(x) = \frac{1}{2}x^2 + \frac{5}{2}x - \frac{3}{2} \)
41. \( f(x) = 3x^3 - 12x^2 + 3x \)
42. \( g(x) = 5x(x^2 - 2x - 1) \)
43. \( f(t) = t^3 - 8t^2 + 16t \)
44. \( f(x) = x^4 - x^3 - 30x^2 \)
45. \( g(t) = t^5 - 6t^3 + 9t \)
46. \( f(x) = x^5 + x^3 - 6x \)
47. \( f(x) = 3x^4 + 9x^2 + 6 \)
48. \( f(x) = 2x^4 - 2x^2 - 40 \)
49. \( g(x) = x^3 + 3x^2 - 4x - 12 \)
50. \( f(x) = x^3 - 4x^2 - 25x + 100 \)

GRAPHICAL ANALYSIS In Exercises 51–54, (a) use a graphing utility to graph the function, (b) use the graph to approximate any \( x \)-intercepts of the graph, (c) set \( y = 0 \) and solve the resulting equation, and (d) compare the results of part (c) with any \( x \)-intercepts of the graph.

51. \( y = 4x^3 - 20x^2 + 25x \)
52. \( y = 4x^3 + 4x^2 - 8x - 8 \)

53. \( y = x^5 - 5x^3 + 4x \)
54. \( y = \frac{1}{4}x^3(x^2 - 9) \)

55–64. See margin.
In Exercises 55–64, find a polynomial function that has the given zeros. (There are many correct answers.)

55. \( 0, 8 \)
56. \( 0, -7 \)
57. \( 2, -6 \)
58. \( -4, 5 \)
59. \( 0, -4, -5 \)
60. \( 0, 1, 10 \)
61. \( 4, -3, 3, 0 \)
62. \( -2, -1, 0, 1, 2 \)
63. \( 1 + \sqrt{3}, 1 - \sqrt{3} \)
64. \( 2, 4 + \sqrt{5}, 4 - \sqrt{5} \)

65–74. See margin.
In Exercises 65–74, find a polynomial of degree \( n \) that has the given zero(s). (There are many correct answers.)

Zero(s) 

<table>
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<th>Zero(s)</th>
<th>Degree</th>
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<tr>
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<tr>
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<td>( n = 3 )</td>
</tr>
<tr>
<td>[ 9 ]</td>
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<tr>
<td>[ 0, -4 ]</td>
<td>( n = 5 )</td>
</tr>
<tr>
<td>[ -1, 4, 7, 8 ]</td>
<td>( n = 5 )</td>
</tr>
</tbody>
</table>

75–88. See margin.
In Exercises 75–88, sketch the graph of the function by (a) applying the Leading Coefficient Test, (b) finding the zeros of the polynomial, (c) plotting sufficient solution points, and (d) drawing a continuous curve through the points.

75. \( f(x) = x^3 - 25x \)
76. \( g(x) = x^4 - 9x^2 \)
77. \( f(t) = \frac{1}{2}(t^2 - 2t + 15) \)
78. \( g(x) = -x^2 + 10x - 16 \)
79. \( f(x) = x^3 - 2x^2 \)
80. \( f(x) = 8 - x^3 \)
81. \( f(x) = 3x^3 - 15x^2 + 18x \)
82. \( f(x) = -4x^3 + 4x^2 + 15x \)
83. \( f(x) = -5x^2 - x^3 \)
84. \( f(x) = -48x^2 + 3x \)
85. \( f(x) = x^3 - 4 \)
86. \( h(x) = \frac{1}{3}x^3(x - 4)^2 \)
87. \( g(t) = -(t - 2)^2(t + 2)^2 \)
88. \( g(x) = \frac{1}{10}(x + 1)^2(x - 3)^3 \)

In Exercises 89–92, use a graphing utility to graph the function. Use the zero or root feature to approximate the real zeros of the function. Then determine the multiplicity of each zero. 89–92. See margin.

89. \( f(x) = x^3 - 16x \)
90. \( f(x) = \frac{1}{2}x^4 - 2x^2 \)
91. \( g(x) = \frac{2}{3}(x + 1)^2(x - 3)(2x - 9) \)
92. \( h(x) = \frac{1}{3}(x + 2)^3(x - 5)^2 \)