2.3 EXERCISES

VOCABULARY

1. Two forms of the Division Algorithm are shown below. Identify and label each term or function.

\( f(x) = d(x)q(x) + r(x) \)

\( \frac{f(x)}{d(x)} = q(x) + \frac{r(x)}{d(x)} \)

- \( f(x) \): dividend; \( d(x) \): divisor;
- \( q(x) \): quotient; \( r(x) \): remainder

In Exercises 2–6, fill in the blanks.

2. The rational expression \( p(x)/q(x) \) is called ______ improper ______ if the degree of the numerator is greater than or equal to that of the denominator, and is called ______ proper ______ if the degree of the numerator is less than that of the denominator.

3. In the Division Algorithm, the rational expression \( f(x)/d(x) \) is ______ improper ______ because the degree of \( f(x) \) is greater than or equal to the degree of \( d(x) \).

4. An alternative method to long division of polynomials is called ______ synthetic division ______, in which the divisor must be of the form \( x - k \).

5. The ______ Factor ______ Theorem states that a polynomial \( f(x) \) has a factor \((x - k)\) if and only if \( f(k) = 0 \).

6. The ______ ______ Theorem states that if a polynomial \( f(x) \) is divided by \( x - k \), the remainder is \( r = f(k) \). Remainder

SKILLS AND APPLICATIONS

ANALYTICAL ANALYSIS In Exercises 7 and 8, use long division to verify that \( y_1 = y_2 \). 7–8. Answers will vary.

7. \( y_1 = \frac{x^2}{x + 2} \). \( y_2 = x - 2 + \frac{4}{x + 2} \)

8. \( y_1 = \frac{x^4 - 3x^2 - 1}{x^2 + 5} \), \( y_2 = x^2 - 8 + \frac{39}{x^2 + 5} \)

GRAPHICAL ANALYSIS In Exercises 9 and 10, (a) use a graphing utility to graph the two equations in the same viewing window, (b) use the graphs to verify that the expressions are equivalent, and (c) use long division to verify the results algebraically. 9–10. See margin.

9. \( y_1 = \frac{x^2 + 2x - 1}{x + 3} \), \( y_2 = x - 1 + \frac{2}{x + 3} \)

10. \( y_1 = \frac{x^4 + x^2 - 1}{x^2 + 1} \), \( y_2 = x^2 - \frac{1}{x^2 + 1} \)

In Exercises 11–26, use long division to divide.

11. \((2x^2 + 10x + 12)/(x + 3) = 2x + 4 \), \( x \neq -3 \)

12. \((5x^2 - 17x - 12)/(x - 4) = 5x + 3 \), \( x \neq 4 \)

13. \((4x^3 - 7x^2 - 11x + 5)/(4x + 5) \) 13–26. See margin.

14. \((6x^3 - 16x^2 + 17x - 6)/(3x - 2) \)

15. \((x^4 + 5x^3 + 6x^2 - x - 2)/(x + 2) \)

16. \((x^3 + 4x^2 - 3x - 12)/(x - 3) \)

17. \((x^3 - 27)/(x - 3) \)

18. \((x^3 + 125)/(x + 5) \)

19. \((7x + 3)/(x + 2) \)

20. \((8x - 5)/(2x + 1) \)

21. \((x^2 - 9)/(x^2 + 1) \)

22. \((x^2 + 7)/(x^3 - 1) \)

23. \((3x + 2x^3 - 9 - 8x^2)/(x^2 + 1) \)

24. \((5x^3 - 16 - 20x + x^4)/(x^2 - x - 3) \)

25. \(\frac{x^4}{(x - 1)^3} \)

26. \(\frac{2x^3 - 4x^2 - 15x + 5}{(x - 1)^2} \)

27–46. See margin. In Exercises 27–46, use synthetic division to divide.

27. \((3x^3 - 17x^2 + 15x - 25)/(x - 5) \)

28. \((5x^3 + 18x^2 + 7x - 6)/(x + 3) \)

29. \((6x^3 + 7x^2 - x + 26)/(x - 3) \)

30. \((2x^3 + 14x^2 - 20x + 7)/(x + 6) \)

31. \((4x^3 - 9x + 8x^2 - 18)/(x + 2) \)

32. \((9x^3 - 16x - 18x^2 + 32)/(x - 2) \)

33. \((-x^3 + 75x - 250)/(x + 10) \)

34. \((3x^3 - 16x^2 - 72)/(x - 6) \)

35. \((5x^3 - 6x^2 + 8)/(x - 4) \)

36. \((5x^3 + 6x + 8)/(x + 2) \)

37. \(\frac{10x^3 - 50x^2 - 800}{x - 6} \)

38. \(\frac{x^5 - 13x^4 - 120x + 80}{x + 3} \)

39. \(\frac{x^3 + 512}{x + 8} \)

40. \(\frac{x^3 - 729}{x - 9} \)

41. \(-\frac{3x^4}{x - 2} \)

42. \(-\frac{3x^4}{x + 2} \)

43. \(\frac{180x - x^4}{x - 6} \)

44. \(\frac{5 - 3x + 2x^2 - x^3}{x + 1} \)

45. \(\frac{4x^3 + 16x^2 - 23x - 15}{x + \frac{1}{2}} \)

46. \(\frac{3x^3 - 4x^2 + 5}{x - \frac{3}{5}} \)
In Exercises 47–54, write the function in the form 
\( f(x) = (x - k)q(x) + r \) for the given value of \( k \), and demonstrate that \( f(k) = r \). 47–54. See margin.

47. \( f(x) = x^3 - x^2 - 14x + 11, \ k = 4 \)
48. \( f(x) = x^3 - 5x^2 - 11x + 8, \ k = -2 \)
49. \( f(x) = 15x^4 + 10x^3 - 6x^2 + 14, \ k = -\frac{2}{3} \)
50. \( f(x) = 10x^3 - 22x^2 - 3x + 4, \ k = \frac{1}{4} \)
51. \( f(x) = x^3 + 3x^2 - 2x - 14, \ k = \sqrt{2} \)
52. \( f(x) = x^3 + 2x^2 - 5x - 4, \ k = -\sqrt{5} \)
53. \( f(x) = -4x^3 + 6x^2 + 12x + 4, \ k = 1 - \sqrt{3} \)
54. \( f(x) = -3x^3 + 8x^2 + 10x - 8, \ k = 2 + \sqrt{2} \)

In Exercises 55–58, use the Remainder Theorem and synthetic division to find each function value. Verify your answers using another method.

55. \( f(x) = 2x^3 - 7x + 3 \)
   (a) \( f(1) \) (b) \( f(-2) \) (c) \( f\left(\frac{1}{2}\right) \) (d) \( f(2) \) See margin.
56. \( g(x) = 2x^6 + 3x^4 - x^2 + 3 \)
   (a) \( g(2) \) (b) \( g(1) \) (c) \( g(3) \) (d) \( g(-1) \) See margin.
57. \( h(x) = x^3 - 5x^2 - 7x + 4 \)
   (a) \( h(3) \) (b) \( h(-2) \) (c) \( h(-2) \) (d) \( h(-5) \) See margin.
58. \( f(x) = 4x^4 - 16x^3 + 7x^2 + 20 \)
   (a) \( f(1) \) (b) \( f(-2) \) (c) \( f(5) \) (d) \( f(-10) \)
   \[ 240 \quad 695 \quad 56,720 \]

In Exercises 59–66, use synthetic division to show that \( x \) is a solution of the third-degree polynomial equation, and use the result to factor the polynomial completely. List all real solutions of the equation. 59–66. See margin.

59. \( x^3 - 7x + 6 = 0 \), \( x = 2 \)
60. \( x^3 - 28x - 48 = 0 \), \( x = -2 \)
61. \( 2x^3 - 15x^2 + 27x - 10 = 0 \), \( x = \frac{1}{2} \)
62. \( 48x^3 - 80x^2 + 41x - 6 = 0 \), \( x = 2 \)
63. \( x^3 + 2x^2 - 3x - 6 = 0 \), \( x = \sqrt{3} \)
64. \( x^3 + 2x^2 - 2x - 4 = 0 \), \( x = \sqrt{2} \)
65. \( x^3 - 3x^2 + 2 = 0 \), \( x = 1 + \sqrt{3} \)
66. \( x^3 - x^2 - 13x - 3 = 0 \), \( x = 2 - \sqrt{5} \)

In Exercises 67–74, (a) verify the given factors of the function \( f \), (b) find the remaining factor(s) of \( f \), (c) use your results to write the complete factorization of \( f \), (d) list all real zeros of \( f \), and (e) confirm your results by using a graphing utility to graph the function. 67–74. See margin.

<table>
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<tr>
<th>Function</th>
<th>Factors</th>
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<td>67. ( f(x) = 2x^3 + x^2 - 5x + 2 )</td>
<td>( (x + 2), (x - 1) )</td>
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<tr>
<td>68. ( f(x) = 3x^3 + 2x^2 - 19x + 6 )</td>
<td>( (x + 3), (x - 2) )</td>
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<tr>
<td>69. ( f(x) = x^4 - 4x^3 - 15x^2 + 58x - 40 )</td>
<td>( (x - 5), (x + 4) )</td>
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**GRAPHICAL ANALYSIS** In Exercises 75–80, (a) use the zero or root feature of a graphing utility to approximate the zeros of the function accurate to three decimal places, (b) determine one of the exact zeros, and (c) use synthetic division to verify your result from part (b), and then factor the polynomial completely. 75–80. See margin.

75. \( f(x) = x^3 - 2x^2 - 5x + 10 \)
76. \( g(x) = x^3 - 4x^2 - 2x + 8 \)
77. \( h(t) = t^3 - 2t^2 - 7t + 2 \)
78. \( f(s) = s^3 - 12s^2 + 40s - 24 \)
79. \( h(x) = x^5 - 7x^4 + 10x^3 - 14x^2 - 24x \)
80. \( g(x) = 6x^4 - 11x^3 - 51x^2 + 99x - 27 \)

In Exercises 81–84, simplify the rational expression by using long division or synthetic division. 81–84. See margin.

81. \( \frac{4x^3 - 8x^2 + x + 3}{2x - 3} \)
82. \( \frac{x^3 + x^2 - 64x - 64}{x + 8} \)
83. \( \frac{x^4 + 6x^3 + 11x^2 + 6x}{x^2 + 3x + 2} \)
84. \( \frac{x^4 + 9x^3 - 5x^2 - 36x + 4}{x^2 - 4} \)

**DATA ANALYSIS: HIGHER EDUCATION** The amounts \( A \) (in billions of dollars) donated to support higher education in the United States from 2000 through 2007 are shown in the table, where \( t \) represents the year, with \( t = 0 \) corresponding to 2000.

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