2.4

VOCABULARY

1. Match the type of complex number with its definition.
   (a) Real number
   (i) $a + bi, \ a \neq 0, \ b \neq 0$
   (b) Imaginary number
   (ii) $a + bi, \ a = 0, \ b \neq 0$
   (c) Pure imaginary number
   (iii) $a + bi, \ b = 0$

In Exercises 2–4, fill in the blanks.

2. The imaginary unit $i$ is defined as $i = \sqrt{-1}$, where $i^2 = \underline{-1}$.
3. If $a$ is a positive number, the principal square root of the negative number $-a$ is defined as $\sqrt{-a} = \sqrt{a}i$.
4. The numbers $a + bi$ and $a - bi$ are called complex conjugates and their product is a real number $a^2 + b^2$.

SKILLS AND APPLICATIONS

In Exercises 5–8, find real numbers $a$ and $b$ such that the equation is true. 5–6. See margin.

5. $a + bi = -12 + 7i$
6. $a + bi = 13 + 4i$
7. $(a - 1) + (b + 3)i = 5 + 8i$ $a = 6, \ b = 5$
8. $(a + 6) + 2bi = 6 - 5i$ $a = 0, \ b = -\frac{5}{2}$

In Exercises 9–20, write the complex number in standard form.

9. $8 + \sqrt{-25}$ $8 + 5i$
10. $5 + \sqrt{-36}$ $5 + 6i$
11. $2 - \sqrt{-27}$ $-2 - 3\sqrt{3}i$
12. $1 + \sqrt{-8}$ $1 + 2\sqrt{2}i$
13. $\sqrt{-80}$ $4\sqrt{5}i$
14. $\sqrt{-4}$ $2i$
15. $14$ $14$
16. $75$ $-75$
17. $-10i + i^2$ $-1 - 10i$
18. $-4i^2 + 2i$ $4 + 2i$
19. $\sqrt{-0.09}$ $0.3i$
20. $\sqrt{-0.0049}$ $0.07i$

In Exercises 21–30, perform the addition or subtraction and write the result in standard form. 21–22, 24. See margin.

21. $(7 + i) + (3 - 4i)$
22. $(13 - 2i) + (-5 + 6i)$
23. $(9 - i) - (8 - i)$
24. $(3 + 2i) - (6 + 13i)$
25. $(-2 + \sqrt{-8}) + (5 - \sqrt{-50})$ $3 - 3\sqrt{2}i$
26. $(8 + \sqrt{-18}) - (4 + 3\sqrt{2}i)$
27. $13i - (14 - 7i)$ $-14 + 20i$
28. $25 + (-10 + 11i) + 15i$
29. $-\left(\frac{3}{2} + \frac{3}{2}i\right) + \left(\frac{3}{2} + \frac{3}{2}i\right) \frac{1}{6} + \frac{7}{6}i$
30. $(1.6 + 3.2i) + (-5.8 + 4.3i)$ $-4.2 + 7.5i$

In Exercises 31–40, perform the operation and write the result in standard form.

31. $(1 + i)(3 - 2i)$ $5 + i$
32. $(7 - 2i)(3 - 5i)$ $11 - 41i$
33. $12i(1 - 9i)$ $108 + 12i$
34. $-8i(9 + 4i)$ $32 - 72i$
35. $(\sqrt{14} + \sqrt{10}i)(\sqrt{14} - \sqrt{10}i)$ $24$
36. $(\sqrt{3} + \sqrt{15}i)(\sqrt{3} - \sqrt{15}i)$ $18$
37. $(6 + 7i)^2 - 13 + 84i$
38. $(5 - 4i)^2$ $9 - 40i$
39. $(2 + 3i)^2 + (2 - 3i)^2$ $-10$
40. $(1 - 2i)^2 - (1 + 2i)^2$ $-8i$

In Exercises 41–48, write the complex conjugate of the complex number. Then multiply the number by its complex conjugate. 43–44, 48. See margin.

41. $9 + 2i$ $9 - 2i, \ 85$
42. $8 - 10i$ $8 + 10i, \ 164$
43. $-1 - \sqrt{5}i$
44. $-3 + \sqrt{2}i$
45. $\sqrt{-20} - 2\sqrt{5}i, \ 20$
46. $\sqrt{-15} - \sqrt{15}i, \ 15$
47. $\sqrt{6}$ $\sqrt{6}$, $\ 6$
48. $1 + \sqrt{8}$

In Exercises 49–58, write the quotient in standard form.

49. $\frac{3}{i}$ $-3i$
50. $\frac{-14}{2i}$ $7i$
51. $\frac{2}{4 - 5i}$ $\frac{8}{41} + \frac{10}{41}i$
52. $\frac{13}{1 - i}$ $\frac{13}{2} + \frac{13}{2}i$
53. $\frac{5 + i}{5 - i}$ $\frac{12}{13} + \frac{5}{13}i$
54. $\frac{6 - 7i}{1 - 2i}$ $4 + i$
55. $\frac{9 - 4i}{i}$ $-4 - 9i$
56. $\frac{8 + 16i}{2i}$ $8 - 4i$
57. $\frac{3i}{(4 - 5i)^2}$ $-\frac{120}{1681} - \frac{27}{1681}i$
58. $\frac{5i}{(2 + 3i)^2}$ $\frac{60}{169} - \frac{27}{169}i$

In Exercises 59–62, perform the operation and write the result in standard form.

59. $\frac{2}{1 + i}$ $\frac{3}{1 - i}$ $-\frac{1}{2} - \frac{5}{2}i$
60. $\frac{2i}{2 + i}$ $\frac{5}{2 - i}$ $\frac{12}{5} + \frac{9}{5}i$
61. $\frac{i}{3 - 2i}$ $\frac{2i}{3 + 8i}$ $\frac{62}{949} + \frac{297}{949}i$
62. $\frac{1 + i}{i}$ $-\frac{3}{4 - i}$ $\frac{5}{17} - \frac{20}{17}i$
In Exercises 63–68, write the complex number in standard form.

63. \( \sqrt{-6} \cdot \sqrt{-2} = 2\sqrt{3} \)
64. \( \sqrt{-5} \cdot \sqrt{-10} = -5\sqrt{2} \)
65. \( (\sqrt{-15})^2 = -15 \)
66. \( (\sqrt{-75})^2 = -75 \)
67. \( (3 + \sqrt{-5})(7 - \sqrt{-10}) \)
68. \( (2 - \sqrt{-6})^2 \)

67–68. See margin.

In Exercises 69–78, use the Quadratic Formula to solve the quadratic equation. 69–78. See margin.

69. \( x^2 - 2x + 2 = 0 \)
70. \( x^2 + 6x + 10 = 0 \)
71. \( 4x^2 + 16x + 17 = 0 \)
72. \( 9x^2 - 6x + 37 = 0 \)
73. \( 4x^2 + 16x + 15 = 0 \)
74. \( 16t^2 - 4t + 3 = 0 \)
75. \( \frac{3}{2}x^2 - 6x + 9 = 0 \)
76. \( \frac{7}{8}x^2 - \frac{3}{4}x + \frac{5}{16} = 0 \)
77. \( 1.4x^2 - 2x - 10 = 0 \)
78. \( 4.5x^2 - 3x + 12 = 0 \)

In Exercises 79–88, simplify the complex number and write it in standard form.

79. \( -6i^3 + i^2 - 1 + 6i \)
80. \( 4i^2 - 2i^3 - 4 + 2i \)
81. \( -14i^5 - 14i \)
82. \( (-i)^3 i \)
83. \( (\sqrt{-72})^3 = -432\sqrt{2}i \)
84. \( (\sqrt{-2})^6 = 8 \)
85. \( \frac{1}{i^3} \)
86. \( \frac{1}{2(i^3)} \) \( \frac{1}{8} \)
87. \( (3i)^4 = 81 \)
88. \( -(i)^6 = -1 \)

89. IMPEDANCE The opposition to current in an electrical circuit is called its impedance. The impedance \( z \) in a parallel circuit with two pathways satisfies the equation

\[
\frac{1}{z} = \frac{1}{z_1} + \frac{1}{z_2}
\]

where \( z_1 \) is the impedance (in ohms) of pathway 1 and \( z_2 \) is the impedance of pathway 2. See margin.

(a) The impedance of each pathway in a parallel circuit is found by adding the impedances of all components in the pathway. Use the table to find \( z_1 \) and \( z_2 \).

(b) Find the impedance \( z \).

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Resistor</th>
<th>Inductor</th>
<th>Capacitor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Impedance</td>
<td>( a )</td>
<td>( bi )</td>
<td>( -ci )</td>
</tr>
</tbody>
</table>

90. Cube each complex number.
(a) \( 2 \) \( 8 \)
(b) \(-1 + \sqrt{3}i \) \( 8 \)
(c) \(-1 - \sqrt{3}i \) \( 8 \)

91. Raise each complex number to the fourth power.
(a) \( 2 \) \( 16 \)
(b) \(-2 - 16 \)
(c) \( 2i \) \( 16 \)
(d) \(-2i \) \( 16 \)

92. Write each of the powers of \( i \) as \( i, -i, 1, \) or \(-1, (a) i^{40} \) (b) \( i^{25} \) \( i \) (c) \( i^{50} - 1 \) (d) \( i^{67} - i \)

EXPLORATION

93–96. See margin.

TRUE OR FALSE? In Exercises 93–96, determine whether the statement is true or false. Justify your answer.

93. There is no complex number that is equal to its complex conjugate.
94. \( -i\sqrt{6} \) is a solution of \( x^4 - x^2 + 14 = 56 \).
95. \( i^{14} + i^{150} - i^{74} - i^{109} + i^{61} = -1 \)
96. The sum of two complex numbers is always a real number.

97. PATTERN RECOGNITION Complete the following.

\begin{align*}
i^1 &= i \\
i^2 &= -1 \\
i^3 &= -i \\
i^4 &= 1 \\
i^5 &= 0 \\
i^6 &= 1 \\
i^7 &= 0 \\
i^8 &= -1 \\
i^9 &= -i \\
i^{10} &= i \\
i^{11} &= 0 \\
i^{12} &= 1
\end{align*}

What pattern do you see? Write a brief description of how you would find \( i \) raised to any positive integer power. See margin.

98. CAPSTONE Consider the functions \( f(x) = 2(x - 3)^2 - 4 \) and \( g(x) = -2(x - 3)^2 - 4 \).

(a) Without graphing either function, determine whether the graph of \( f \) and the graph of \( g \) have \( x \)-intercepts. Explain your reasoning.
(b) Solve \( f(x) = 0 \) and \( g(x) = 0 \).
(c) Explain how the zeros of \( f \) and \( g \) are related to whether their graphs have \( x \)-intercepts.
(d) For the function \( f(x) = (a(x - h))^2 + k \), make a general statement about how \( a, h, \) and \( k \) affect whether the graph of \( f \) has \( x \)-intercepts, and whether the zeros of \( f \) are real or complex.

99. ERROR ANALYSIS Describe the error. See margin.

\[
\sqrt{-6} \cdot \sqrt{-6} = \sqrt{(-6)(-6)} = \sqrt{36} = 6
\]

100. PROOF Prove that the complex conjugate of the product of two complex numbers \( a_1 + b_1i \) and \( a_2 + b_2i \) is the product of their complex conjugates.

Proof

\[
\text{Prove: } (a_1 + b_1i)(a_2 + b_2i) = (a_1 \cdot a_2 - b_1 \cdot b_2) + (a_1 \cdot b_2 + b_1 \cdot a_2)i
\]

101. PROOF Prove that the complex conjugate of the sum of two complex numbers \( a_1 + b_1i \) and \( a_2 + b_2i \) is the sum of their complex conjugates.

Proof