Bellwork: 9/28/16

1. Which of the following expressions is equivalent to $\frac{1}{2}y^2(6x + 2y + 12x - 2y)$?
   a. $9y^2$
   b. $18xy$
   c. $3xy^2 + 12x$
   d. $9y^2 - 2y^3$

2. If $a$, $b$, and $c$ are positive integers such that $a^9 = x$ and $c^9 = y$ then $xy = ?$
   f. $ac^9$
   h. $(ac)^9$
   k. $(ac)^{9\text{a}}$
   g. $ac^{2\text{a}}$
   j. $(ac)^{3\text{b}}$

Homework Quiz #2:

p. 164
11. $2\sqrt[3]{3}i$
19. $z_1 = 9 + 16i, z_2 = 20 - 10i$
22. $8 + 4i$
92. a. 1, b. i, c. -1, d. -i
23. 1
94. true: plug in $56 = 56$
32. $11 - 4i$
95. false: $1 - 1 + 1 - i + i = 1$
42. $8 - 10i, 164$
53. $12/13 + 5/13i$
72. $1/2 \pm 2i$
78. $1/3 \pm \sqrt{2}i$

Sep 30-8:16 AM

2.5 Zeros of Polynomial Functions

Objective: SWBAT find complex zeros of and find real zeros of polynomial functions using the rational root theorem.

Vocabulary:

**Fundamental Theorem of Algebra:** If $f(x)$ is a polynomial of degree $n$, where $n > 0$, then $f$ has at least one zero in the complex number system.

**Linear Factorization Theorem:** If $f(x)$ is a polynomial function of degree $n$, $n > 0$, then $f$ has precisely $n$ linear factors.
Zeros of a Polynomial Functions

**Raional Root Test:**

Every raional zero of \( f \) has the form,

\[
\text{raional zero } = \frac{p}{q} = \frac{\text{factors of constant term}}{\text{factors of leading coefficient}}
\]

\( p \) = a factor of the constant term \( a_0 \)

\( q \) = a factor of the leading coefficient \( a_n \)

**Example 1:** Use the Raional Root Test to list all possible raional zeros of \( f \). Verify that the zeros of \( f \) shown on the graph provided are contained in the list:

\[ f(x) = ax^2 - 4x + 4 \]

\[ f(x) = 2x^3 - 8x^2 + 3 \]

\[ p = \frac{1}{1}, \frac{1}{2}, \frac{1}{4} \]

\[ q = \frac{1}{1}, \frac{1}{2}, \frac{1}{4} \]

\[ p = \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \]

\[ q = \frac{1}{2}, \frac{1}{4}, \frac{1}{8} \]

**Example 1:**

Use the Raional Zero Test to list all possible raional zeros of \( f \). Verify that the zeros of \( f \) shown on the graph provided are contained in the list:

\[ f(x) = 2x^3 - 5x + 2 \]

\[ f(x) = 2x^3 + x^2 - 5x + 10 \]

\[ p = \frac{1}{1}, \frac{1}{2}, \frac{1}{5} \]

\[ q = \frac{1}{1}, \frac{1}{2}, \frac{1}{5} \]

\[ p = \frac{1}{2}, \frac{1}{5}, \frac{1}{10} \]

\[ q = \frac{1}{2}, \frac{1}{5}, \frac{1}{10} \]
2.5 Zeros of Polynomials

Example 3:
List all possible rational zeros of \( f(x) = x^3 - 3x^2 - 2x + 4 \). Then determine which, if any, are zeros.

\[
\begin{array}{c|cc|c}
\text{P} & 1 & 2 & 4 \\
\text{Q} & 1 & -3 & -2 & 4 \\
\hline
1 & -2 & -4 & 0 \\
\hline
\end{array}
\]

\[
x = \frac{2 \pm \sqrt{4 - 4(1)(-4)}}{2} = \frac{2 \pm 2\sqrt{5}}{2}
\]

List all possible rational zeros of \( f(x) = x^3 - 6x^2 + 11x - 6 \). Then determine which, if any, are zeros.

\[
\begin{array}{c|cc|c}
\text{P} & 1 & 2 & 2 & 1 \\
\text{Q} & 1 & -6 & 11 & -6 \\
\hline
1 & -5 & 6 & 0 \\
\hline
\end{array}
\]

Descartes' Rule of Signs: gives us information about the number of positive and negative real zeros of a polynomial function by looking at a polynomial's variations in sign.

- Number of
- Number of

Example 4:
Describe the possible real zeros of \( f(x) = x^4 - 3x^3 - 5x^2 + 2x + 7 \).

Example 5:
Describe the possible real zeros of \( g(x) = -x^3 + 8x^2 - 7x + 9 \).

Conjugate Root Theorem: when a polynomial equation in one variable with real coefficients had a root of the form \( a + bi \), where \( b \neq 0 \), then its complex conjugate, \( a - bi \), is also a root.

Example 5:
Write a polynomial of least degree with real coefficients in standard form that has the given zeros.

\(-4, -2, -2i\)

\[
\begin{align*}
(x+4)(x+2)(x+2i)(x-2i) \\
x^2 - 2(1)(x + 2i)(x - 2i) \\
(x+4)(x-2i)(x+2i) \\
(x+2)(x^2+4) \\
x^4 + 2x^2 + 4x^2 + 8x^2 + 4 + 8 + 16x + 32 \\
x^4 + 2x^3 + 12x^2 + 24x + 32
\end{align*}
\]
Example 6:
Find all complex zeros of $h(x) = x^4 + x^3 - 3x^2 + 9x - 108$ given that $x = -3i$ is a zero of $h$.

2.5 assignment
pg 176 # 10, 16, 18, 20, 22, 26, 30, 38, 46, 55, 88, 92

\[ \# 10, 16, 18, 20, 22, 30, 38, 70, 71 \]