3.4 **EXERCISES**


**VOCABULARY:** Fill in the blanks.

1. To **solve** an equation in \( x \) means to find all values of \( x \) for which the equation is true.
2. To solve exponential and logarithmic equations, you can use the following One-to-One and Inverse Properties.
   (a) \( a^x = a^y \) if and only if \( x = y \).
   (b) \( \log_a x = \log_a y \) if and only if \( x = y \).
   (c) \( a^{\log_a x} = x \).
   (d) \( \log_a a^x = x \).
3. To solve exponential and logarithmic equations, you can use the following strategies.
   (a) Rewrite the original equation in a form that allows the use of the One-to-One Properties of exponential or logarithmic functions. **One-to-One**
   (b) **logarithmic; logarithmic** Rewrite an exponential equation in exponential form and apply the Inverse Property of exponential functions.
   (c) **logarithmic; logarithmic** Rewrite a logarithmic equation in exponential form and apply the Inverse Property of exponential functions.
4. An **exponential** solution does not satisfy the original equation. **extraneous**

**SKILLS AND APPLICATIONS**

In Exercises 5–12, determine whether each \( x \)-value is a solution (or an approximate solution) of the equation.

5. \( 4x^2 = 64 \)
   (a) \( x = 5 \) **Yes**
   (b) \( x = 2 \) **No**

6. \( 2^{3x+1} = 32 \)
   (a) \( x = -1 \) **No**
   (b) \( x = 2 \) **No**

7. \( 3e^{x^2} = 75 \)
   (a) \( x = -2 + e^{25} \) **No**
   (b) \( x = -2 + \ln 25 \) **Yes**
   (c) \( x \approx 1.219 \) **Yes, approx.**

8. \( 4e^{-x} = 60 \)
   (a) \( x = 1 + \ln 15 \) **Yes**
   (b) \( x = 3.708 \) **Yes, approx.**
   (c) \( x = \ln 16 \) **No**

9. \( \log_4(3x) = 3 \)
   (a) \( x \approx 21.333 \) **Yes, approx.**
   (b) \( x = -4 \) **No**
   (c) \( x = \frac{64}{3} \) **Yes**

10. \( \log_2(x + 3) = 10 \)
    (a) \( x = 1021 \) **Yes**
    (b) \( x = 17 \) **No**
    (c) \( x = 10^2 - 3 \) **No**

11. \( \ln(2x + 3) = 5.8 \)
    (a) \( x = \frac{1}{2}(-3 + \ln 5.8) \) **Yes**
    (b) \( x = 45.701 \) **Yes, approx.**
    (c) \( x \approx 163.650 \) **Yes, approx.**

In Exercises 13–24, solve for \( x \).

13. \( 4^x = 16 \)
   (a) \( x = 2 \) **Yes**
   (b) \( x = 3 \) **No**

14. \( 3^x = 243 \)
   (a) \( x = 5 \) **Yes**
   (b) \( x = -3 \) **No**

15. \( \sqrt[3]{x} = 32 \)
   (a) \( x = 2 \) **Yes**
   (b) \( x = 0 \) **No**

16. \( \sqrt[3]{x} = \sqrt[3]{64} \)
   (a) \( x = 6 \) **Yes**
   (b) \( x = -3 \) **Yes, approx.**

17. \( \ln(x - 2) = 0 \)
   (a) \( x = 2 \) **Yes**
   (b) \( x = 0 \) **No**

18. \( \ln(x - 5) = 0 \)
   (a) \( x = 5 \) **Yes**
   (b) \( x = -5 \) **No**

19. \( e^{2x} = 2 \)
   (a) \( x = 0.693 \) **No**
   (b) \( x = 1 \) **Yes**

20. \( e^{x^2} = 1.4 \)
   (a) \( x = 1 \) **Yes**
   (b) \( x = 0 \) **No**

21. \( \log_2(x - 1) = 2 \)
   (a) \( x = 4 \) **Yes**
   (b) \( x = 1 \) **No**

22. \( \log_3(x + 2) = 3 \)
   (a) \( x = 1 \) **Yes**
   (b) \( x = -5 \) **No**

In Exercises 25–28, approximate the point of intersection of the graphs of \( f \) and \( g \). Then solve the equation \( f(x) = g(x) \) algebraically to verify your approximation.

25. \( f(x) = 2^x \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 8 \) **No**

26. \( f(x) = 27^x \)
   (a) \( x = \frac{3}{3} \) **Yes**
   (b) \( x = 9 \) **No**

27. \( f(x) = \log_3(x) \)
   (a) \( x = 27 \) **Yes**
   (b) \( x = 3 \) **No**

28. \( f(x) = \ln(x - 4) \)
   (a) \( x = 5 \) **Yes**
   (b) \( x = 0 \) **No**

In Exercises 29–70, solve the exponential equation algebraically. Approximate the result to three decimal places.

29. \( e^x = e^{x^2} - 2 \)
   (a) \( x = 2 \) **Yes**
   (b) \( x = -1 \) **No**

30. \( e^{2x} = e^{x^2} - 2 \)
   (a) \( x = 4 \) **Yes**
   (b) \( x = 0 \) **No**

31. \( e^{x^3} = e^{x^2} - 3 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 0 \) **No**

32. \( e^{x^2} = e^{x^3} - 3 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 0 \) **No**

33. \( 4^{3x} = 20 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

34. \( 2(5^x) = 32 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

35. \( 2e^x = 10 \)
   (a) \( x = 5 \) **Yes**
   (b) \( x = 4.609 \) **No**

36. \( e^{x^3} = 91 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 2 \) **No**

37. \( e^{x^2} - 9 = 19 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

38. \( e^{x^2} + 10 = 47 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

39. \( 2^x = 80 \)
   (a) \( x = 6 \) **Yes**
   (b) \( x = 5 \) **No**

40. \( 2^{x^2} = 3000 \)
   (a) \( x = 5 \) **Yes**
   (b) \( x = 4 \) **No**

41. \( 5^{-x} = 0.2 \)
   (a) \( x = 2 \) **Yes**
   (b) \( x = 3 \) **No**

42. \( 4^{-x} = 0.10 \)
   (a) \( x = 2 \) **Yes**
   (b) \( x = 3 \) **No**

43. \( 3^{-x} = 27 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 0 \) **No**

44. \( 2^{-x} = 32 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

45. \( 3^{-x} = 565 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

46. \( 8^{-x} = 431 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

47. \( 3 - \frac{\ln 565}{\ln 2} \approx -6.142 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**

48. \( -\ln 64 - \ln 431 = -4.917 \)
   (a) \( x = 3 \) **Yes**
   (b) \( x = 1 \) **No**
47. \(8(10^{3x}) = 12\)
48. \(5(10^{-x/6}) = 7\)
49. \(3(5^{x-1}) = 21\)
50. \(8(3^{x-5}) = 40\)
51. \(e^{2x} = 12\)
52. \(e^{2x} = 50\)
53. \(500e^{-x} = 300\)
54. \(1000e^{-4x} = 75\)
55. \(7 - 2e^x = 5\)
56. \(-14 + 3e^x = 11\)
57. \(6(2^{3x-1}) - 7 = 9\)
58. \(8(4^{2x-3}) + 13 = 41\)
59. \(e^{2x} - 4e^x - 5 = 0\)
60. \(e^{2x} - 5e^{x} + 6 = 0\)
61. \(e^{2x} - 3e^x - 4 = 0\)
62. \(e^{2x} + 9e^x + 36 = 0\)
63. \(500 \quad 100e^{-x/2} = 20\)
64. \(400 \quad 1 + e^{-x} = 350\)
65. \(3000 \quad 2 + e^{2x} = 2\)
66. \(e^{6x} - 14 = 7\)
67. \((1 + 0.065)^{365y} = 4\)
68. \((4 - 2.517)^{10y} = 21\)
69. \((1 + 0.125t)^{12y} = 2\)
70. \((16 - 0.878)^{30y}\)

In Exercises 71–80, use a graphing utility to graph and solve the equation. Approximate the result to three decimal places. Verify your result algebraically. 71–80. See margin.

71. \(7 = 2^x\)
72. \(5^r = 212\)
73. \(6e^{1-x} = 25\)
74. \(-4e^{-x} + 15 = 0\)
75. \(3e^{2x/2} = 962\)
76. \(8e^{-2x/3} = 11\)
77. \(e^{0.09y} = 3\)
78. \(-e^{1.8x} + 7 = 0\)
79. \(e^{0.125y} - 8 = 0\)
80. \(e^{2.724x} = 29\)

In Exercises 81–112, solve the logarithmic equation algebraically. Approximate the result to three decimal places.

81. \(\ln x = -3\)
82. \(\ln x = 1.6\)
83. \(\ln x - 7 = 0\)
84. \(\ln x + 1 = 0\)
85. \(\ln 2x = 2.4\)
86. \(2.1 = \ln 6x\)
87. \(\log x = 6\)
88. \(\log 3z = 2\)
89. \(3 \ln 5x = 10\)
90. \(2 \ln x = 7\)
91. \(\ln \sqrt{x} + 2 = 1\)
92. \(\ln \sqrt{x} - 8 = 5\)
93. \(7 + 3 \ln x = 5\)
94. \(2 - 6 \ln x = 10\)
95. \(-2 + 2 \ln 3x = 17\)
96. \(2.3 + 3 \ln x = 12\)
97. \(6 \log_{10}(0.5x) = 11\)
98. \(4 \log_{10}(x-6) = 11\)
99. \(\ln x - \ln(x+1) = 2\)
100. \(\ln x + \ln(x+1) = 1\)
101. \(\ln x + \ln(x-2) = 1\)
102. \(\ln x + \ln(x+3) = 1\)
103. \(\ln(5x) = \ln(x-1) - \ln(x+1)\)
104. \(\ln(x+1) - \ln(x-2) = \ln x\)
105. \(\log_{10}(2x - 3) = \log_{10}(x + 4)\)
106. \(\log_{10}(3x + 4) = \log_{10}(x - 10)\)
107. \(\log_{10}(x + 4) - \log_{10}(x - 10)\)
108. \(\log_{10}(2x + 3) = \log_{10}(x - 10)\)
109. \(\log_{10}(x - 3) = \log_{10}(x - 10)\)
110. \(\log_{10}(x + 3) = \log_{10}(x - 10)\)
111. \(\log_{10}(x + 3) = \log_{10}(x - 10)\)
112. \(\log_{10}(x + 3) = \log_{10}(x - 10)\)

In Exercises 113–116, use a graphing utility to graph and solve the equation. Approximate the result to three decimal places. Verify your result algebraically. 113–116. See margin.

113. \(3 - \ln x = 0\)
114. \(10 - 4 \ln(x - 2) = 0\)
115. \(2 \ln(x + 3) = 3\)
116. \(\ln(x + 1) = 2 - \ln x\)

**COMPOUND INTEREST** In Exercises 117–120, \$250 is invested in an account at interest rate \(r\), compounded continuously. Find the time required for the amount to (a) double and (b) triple. 117–118. See margin.

117. \(r = 0.05\)
118. \(r = 0.045\)
119. \(r = 0.025\)
120. \(r = 0.0375\)
(a) \(27.73\) yr (b) \(43.94\) yr
119. 120. (a) \(18.48\) yr (b) \(29.30\) yr

In Exercises 121–128, solve the equation algebraically. Round the result to three decimal places. Verify your answer using a graphing utility.

121. \(2x^2 \log_2(x) + 2x^2 \log_2(3x) = 0\)
122. \(-x^2 \log_2(x) + 2x \log_2(3x) = 0\)
123. \(-x^2 \log_2(x) + 2x \log_2(3x) = 0\)
124. \(e^{-x} + 2e^{-2x} = 0\)
125. \(2x \ln x + x = 0\)
126. \(\ln(x^2)\) e^{-x/2} = 0.607
127. \(\ln x^2 = 0\)
128. \(2x \ln(\frac{1}{x}) = 0\)
129. \(\ln(\frac{1}{x}) = 0.368\)
130. \(\ln^{-1/2} = 0.607\)

**DEMAND** The demand equation for a limited edition coin set is

\[ p = 1000 \left(1 - \frac{5}{5 + e^{-0.001x}}\right) \]

(a) \(210\) coins
(b) \(588\) coins

Find the demand \(x\) for a price of (a) \(p = \$139.50\) and (b) \(p = \$99.99\).

130. **DEMAND** The demand equation for a hand-held electronic organizer is

\[ p = 5000 \left(1 - \frac{4}{4 + e^{-0.002x}}\right) \]

(a) \(303\) units
(b) \(528\) units

Find the demand \(x\) for a price of (a) \(p = \$600\) and (b) \(p = \$400\).