Section: 4.7 Law of Sines

Objective: SWBAT find measures of all angles and side lengths of any triangle.
General Comments

We learned to solve right triangles in chapter 4. We will start this chapter by learning to solve oblique triangles (non-right triangles).

Please note that angles are Capital letters and the side opposite is the same letter in lower case.
What we already know

• The interior angles total $180^\circ$.
• We can’t use the Pythagorean Theorem. Why not?
• For later, area $= \frac{1}{2} bh$
Playing with the triangle

Let's drop an altitude and call it $h$.

If we think of $h$ as being opposite to both $A$ and $B$, then

$$\sin A = \frac{h}{b} \text{ and } \sin B = \frac{h}{a}$$

Let's solve both for $h$.

$$h = b \sin A \text{ and } h = a \sin B$$

This means

$$b \sin A = a \sin B \text{ and dividing by } ab.$$
If I were to drop an altitude to side $a$, I could come up with

$$\frac{\sin B}{b} = \frac{\sin C}{c}$$

Putting it all together gives us the Law of Sines.

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

You can also use it upside-down.

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$
Objective: SWBAT find measures of all angles and side lengths of any triangle.

**LAW OF SINES**

Non-Right Triangles (Oblique)

If ABC is a triangle with sides a, b, and c, then:

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]
What good is it?

The Law of Sines can be used to solve the following types of oblique triangles

• Triangles with 2 known angles and 1 side (AAS or ASA)
• Triangles with 2 known sides and 1 angle opposite one of the sides (SSA)

With these types of triangles, you will almost always have enough information/data to fill out one of the fractions.

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]
General Process

1. Except for the ASA triangle, you will always have enough information for 1 full fraction and half of another. Start with that to find a fourth piece of data.

2. Once you know 2 angles, you can subtract from 180 to find the 3rd.

3. To avoid rounding error, use given data instead of computed data whenever possible.
Example 1:

Law of Sines works quickly whenever you are given two angles and a side. (AAS or ASA)

Use the given information to solve the triangle.

Ex. 1 \[ C = 102.3^\circ, \ B = 28.7^\circ, \text{ and } b = 27.4 \]
Example 2:

Law of Sines works quickly whenever you are given two angles and a side. (AAS or ASA)

Use the given information to solve the triangle.

Ex. 2  Let $A = 35^\circ$, $B = 10^\circ$, and $c = 45$
With ONE partner....

Turn your book to page 291 and complete “Guided Practice” number 1A.
Math pick up lines!

Hey baby! What's your sine?
Law of Sines

Topic 2

“A Pain in the @SS!”
Example 3 SSA

Let $A = 40^\circ$, $b = 10$, and $a = 7$
With ONE partner....

Turn your book to page 293 and complete “Guided Practice” number 3B.
Summary...

Why should you use given data whenever possible?

Worksheet #'s 1, 2, 3, 4, 7, 8

Unit Circle Quiz!!
Bellwork

Try – from memory – to label the following angles on the unit circle in Degrees.

Objective: SWBAT find measures of all angles and side lengths of any triangle.
Homework Answers
Worksheet 1

#1  A=24.9°, B = 26.1°, a = 24.9 in
#2  A=86°, b = 24 cm, c = 9 cm
#3  A=28.1°, C=117.9°, c = 30 ft.
#4  A=29°, a = 21 ft, c = 42.9 ft
#7 A=25°, a=18 km, c=20 km
#8  A=108°, a = 10 mi, b = 11 mi
A Pain in the Angle Side Side Side

Let’s consider the case where we have an angle, an adjacent side, and an opposite side. For example, I have angle A, side b, and side a.

Sometimes $a$ is too short to reach. You attempt to work this problem like example 3. Your calculator will give you an error message and catch the error.

Sometimes $a$ is just right. It reaches with a right angle. You work this problem like example 3.
Sometimes $a$ is so long it only reaches one way. This problem also works just like example 3.

Sometimes $a$ is just the right length that it can form 2 different triangles. Following example 3 solves the outer triangle. You have to be on the look-out to catch the second triangle.

If $a < b$, then you will get either:

an error, a right angle, or 2 two triangles.
Possible Triangles with @SS

\[ \frac{\sin 40}{6} = \frac{\sin B}{16} \]
\[ 16 \sin 40 = 6 \cdot \sin B \]
\[ \sin B = \frac{16 \sin 40}{6} \]
\[ \sin^{-1} 40 \]

\[ A \]

\[ b = 16 \]

\[ \text{A is acute!!} \]

\[ a < b \text{ and } a < h \]

\[ h = b \sin A \]

\[ h = 16 \sin 40 \]
\[ h = 10.3 \]

\[ \text{NO SOLUTION} \]
Possible Triangles with @SS

A is acute!!

h = b \sin A
h = 10 \sin 36.87
h = 6

ONE SOLUTION

a < b and a = h

A

b = 10

36.87°

h

a = 6

6 < 10

6 = 6
Possible Triangles with @SS

A is acute!!

h = b \sin A

h = 15 \sin 40^

h = 9.64

a < b and a > h

two SOLUTION
Possible Triangles with @SS

A is acute!!

ONE SOLUTION
Possible Triangles with @SS

A is OBTUSE!!

NO SOLUTION
Possible Triangles with @SS

A is OBTUSE!!

ONE SOLUTION
Let $A = 40$, $b = 10$, and $a = 9$. Angle side side

Is $a < b$?  
$9 < 10 \text{ yes}$

Find $h$:  
$h = bsinA$  
$h = 10 \sin 40$  
$h = 6.4$

Is $a > h$?  
$9 > 6.4 \text{ yes}$

2 solutions: Solve large $\triangle$ first
An Example of a Pain in the Angle Side Side

Let $A = 40$, $b = 10$, and $a = 9$. Angle side side

$$\frac{\sin 40}{9} = \frac{\sin B}{10}$$

$$180 - 85.6$$

$$\frac{10 \sin 40}{9} = \frac{9 \sin B}{9}$$

$$\sin B = 10 \sin 40$$

$$\sin^{-1}\left(\frac{10 \sin 40}{9}\right) = B$$

$$B = 45.6^\circ$$

$$\frac{\sin 40}{9} = \frac{\sin 94.4}{c}$$

$$c \sin 40 = 9 \sin 94.4$$

$$\frac{c \sin 40}{\sin 40} = 9 \frac{\sin 94.4}{\sin 40}$$

$$c = 14$$
Finding the Second Triangle

Start by finding $B' = 180 - B$

$B' = 180 - 45.6 \approx 134.4$

$\angle C' = 5.6^\circ$

$\frac{\sin 40}{9} = \frac{\sin 5.6}{c'}$

$c' \approx \frac{9 \sin 5.6}{\sin 40}$

$c' \approx 1.4$

Now solve this triangle.
Summary...

When could your triangle have more than one solution?

Worksheet #’s 5, 6, 10, 11, 13, 15
Another Perspective on Starting the Second Triangle

If the 2 sides are equal, the 2 angles are equal.

Since supplementary angles total 180, \( B' = 180 - B \).

Since \( A \), \( b \), and \( a \) are given, once you know \( B' \) you can finish the triangle on the left.
Contemplating Triangles

Solving like example 3 will give you the purple (outer) triangle.

Since the 2 red lines (a) are equal, the 2 base angles (B) are also equal.

Since B and B' form a straight line, $B' = 180 - B$.

Use B' to compute C'. Subtract A and B' from 180 to find C'.