Bellwork

Decide if each sequence is arithmetic or geometric. Then state the difference or ratio.

G
1, 5, 25, 125,... \( r = 5 \)
\( \frac{5}{1} = 5 \)
\( \frac{25}{5} = 5 \)

A
14, 11, 8, 5, 2,... \( d = -3 \)
\(-3\) \(-3\) \(-3\) \(-3\)
Homework Answers

1. Yes; \( d = 8 \)  

3. yes; \( r = 2 \)

5. -38, -28, -18, -8, 2; 472

7. -2, 6, -18, 54, -162; 4374

9. \( a_n = 2.1n – 9 \); \( a_{30} = 54 \)

11. \( a_n = (-2)6^{n-1} \); \( a_8 = -559872 \)

13. \( d = 5 \); \( a_n = 5n – 37 \)
Chapter 11
Sequences and Series

Day 2:
Intro to Series

Objective:
Calculate sums of series.
Vocabulary

Series – *sum of the terms of a sequence*

- Summation (Sigma) Notation:

\[ \sum_{n=1}^{4} 2^n = 30 \]

\[ a_1 = 2^1 = 2 \]
\[ a_2 = 2^2 = 4 \]
\[ a_3 = 2^3 = 8 \]
\[ a_4 = 2^4 = \frac{16}{30} \]

“The sum of the first 4 terms of the series of \( 2^n \) starting at 1 and ending at 4.”
Examples

• Find the sum of the series:

\[ \sum_{n=1}^{4} 3n - 4 = 14 \]

\[ a_1 = 3(1) - 4 = -1 \]
\[ a_2 = 3(2) - 4 = 2 \]
\[ a_3 = 3(3) - 4 = 5 \]
\[ a_4 = 3(4) - 4 = 8 \]
Examples

• Find the sum of the series:

\[
\sum_{n=1}^{5} \frac{n^2 - 1}{2} = 25
\]

\[
a_1 = \frac{1^2 - 1}{2} = 0
\]

\[
a_2 = \frac{2^2 - 1}{2} = 1.5
\]

\[
a_3 = \frac{3^2 - 1}{2} = 4
\]

\[
a_4 = \frac{4^2 - 1}{2} = 7.5
\]

\[
a_5 = \frac{5^2 - 1}{2} = 12
\]
Arithmetic Series

Sum of the terms of an arithmetic sequence.

Formula or Rule for Arithmetic Series:

\[
S_n = \frac{n}{2}(a_1 + a_n)
\]

The formula gives the sum of a finite arithmetic series.
Example 1

- Given \( a_n = 3n - 2 \)
- find the sum of the first 20 terms.

\[
S_n = \frac{n}{2} (a_1 + a_n)
\]

\[
S_n = \frac{20}{2} (1 + 58)
\]

\[
= 10 (59)
\]

\[
= 590
\]

\[
a_1 = 3(1) - 2 = 1
\]

\[
a_{20} = 3(20) - 2 = 58
\]
Example 2

Consider the arithmetic series \(20 + 18 + 16 + 14\ldots\)

- Find the sum of the first 25 terms.

\[
S_n = \frac{n}{2} (a_1 + a_n)
\]

\[
S_n = \frac{25}{2} (20 + (-28))
\]

\[
= \frac{25}{2} (-8) = -100
\]

\[
a_n = d(n-1) + a_1
\]

\[
a_n = -2(25-1) + 20
\]

\[
a_n = -48 + 20
\]

\[
a_n = -28
\]
Examples

• Find the indicated sum:

\[ \sum_{n=6}^{28} 5n - 17 \]

\[ a_1 = 5(6) - 17 = 13 \]
\[ a_n = 5(28) - 17 = 123 \]

\[ S_n = \frac{n}{2}(a_1 + a_n) \]
\[ S_n = \frac{23}{2}(13 + 123) = 1564 \]

\# of terms = upper bound - lower bound + 1
\[ n = 28 - 6 + 1 \]
\[ n = 23 \]
Geometric Series

Sum of the terms of a Geometric Sequence

Formula or Rule for Geometric Series:

\[ S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right) \]

The formula gives the sum of a finite geometric series.
Example 3

Find the sum of the first 20 terms of the geometric series.

\[
\sum_{n=1}^{20} 4 \cdot 2^{n-1}
\]

\[
S_n = a_1 \left( \frac{1 - r^n}{1 - r} \right)
\]

\[
S_{20} = 4 \left( \frac{1 - 2^{20}}{1 - 2} \right) = 4194300
\]
Example 4

Consider the geometric series 4 + 2 + 1 + \frac{1}{2} \ldots

\begin{align*}
& a_1, \quad 2q = \frac{1}{2} \text{ or } S = \frac{5}{r} = 5 \\
& \text{• Find the sum of the first 10 terms.}
\end{align*}

\begin{align*}
S_n &= a_1 \left( \frac{1-r^n}{1-r} \right) \\
S_n &= 4 \left( \frac{1-\left(\frac{1}{2}\right)^{10}}{1-\frac{1}{2}} \right) \\
\therefore S_n &= 7.992
\end{align*}
Infinite Geometric Series...

Formula or Rule for Infinite Geometric Series:

\[ S = \frac{a_1}{1 - r} \]

Given \(|r| < 1\)

If \(|r| < 1\), then the sum exists and converges.

If \(|r| \geq 1\), the series has no sum, it DIVERGES.
Example 5

- Consider the geometric series. Find the sum if it exists.

\[
\sum_{i=1}^{\infty} 3 \left( \frac{3}{2} \right)^{n-1}
\]

- If \( r \geq 1 \), there is no sum.

\[
\sum_{i=1}^{\infty} 3 \left( 0.7 \right)^{i-1}
\]

- If \( r < 1 \), the sum exists.

S_{n} = \frac{a_{1}}{1-r}

S_{n} = \frac{3}{1-0.7} = 10
Summary...

• What is the difference between a sequence and a series?

• What HAS to be true in order for an infinite geometric series to have a sum?

Series Practice Worksheet 4-18 EVENS