4-4
Factoring Quadratic Expressions

Vocabulary

Review
1. Complete each factor tree.

24

\[ \begin{array}{c}
2 \\
3 \\
\end{array} \]

54

\[ \begin{array}{c}
9 \\
\end{array} \]

Vocabulary Builder

factor (noun) FAK tər

Other Word Forms: factor (verb)

Main Idea: The factors of an expression are similar to the factors of a number.

Definition: The factors of a given expression are expressions whose product equals the given expression. When you factor an expression, you break it into smaller expressions whose product equals the given expression.

Example: The factors of the expression \(2x^2 - x - 10\) are \(2x - 5\) and \(x + 2\).

Use Your Vocabulary

2. Circle the prime factors of \(24xy\).

\[ \begin{array}{c}
24 \cdot x \cdot y \\
2 \cdot 4 \cdot x \cdot y \\
2^3 \cdot 3 \cdot x \cdot y \\
\end{array} \]

3. Circle the prime factors of \(54a^2b\).

\[ \begin{array}{c}
54 \cdot a^2 \cdot b \\
5 \cdot 4 \cdot a^2 \cdot b \\
2 \cdot 3^3 \cdot a^2 \cdot b \\
\end{array} \]
Factoring $ax^2 + bx + c$ when $a = \pm 1$

**Got It?** What is the expression $x^2 + 14x + 40$ in factored form?

4. Complete the factor table. Then circle the pair of factors whose sum is 14.

| Factors of 40 | 1, 40 | 2, | | |
| **Sum of Factors** | | | | |

5. Circle the expression written as the product of two binomials.

\[(x + 1)(x + 40) \quad (x + 2)(x + 20) \quad (x + 4)(x + 10) \quad (x + 5)(x + 8)\]

**Got It?** What is the expression $x^2 - 11x + 30$ in factored form?

6. Underline the correct word(s) to complete each sentence.

I need to find factors that **multiply** / **sum** to 30 and **multiply** / **sum** to $-11$.

At least one of the factors that sum to $-11$ must be **positive** / **negative**.

The two factors that multiply to 30 must both be **positive** / **negative**.

7. Circle the factors of 30 that sum to $-11$.

| 1 and 30 | 2 and 15 | 3 and 10 | 5 and 6 |
| -1 and -30 | -2 and -15 | -3 and -10 | -5 and -6 |

8. Factor the expression.

\[x^2 - 11x + 30 = \left(x \quad \right)\left(x \quad \right)\]

**Got It?** What is the expression $-x^2 + 14x + 32$ in factored form?

9. Rewrite the expression to show a trinomial with a leading coefficient 1.

\[-x^2 + 14x + 32 = \]

10. **Reasoning** You are looking for factors of $-32$ that sum to $-14$. Which of the factors has the greater absolute value, the negative factor or the positive factor? How do you know?

________________________________________________________________________

________________________________________________________________________

11. Circle the factors of $-32$ that sum to $-14$.

| -1 and 32 | -2 and 16 | -4 and 8 |
| 1 and $-32$ | 2 and $-16$ | 4 and $-8$ |

12. Write the factored form of the expression.
Problem 2  Finding Common Factors

**Got It?** What is the expression $7n^2 - 21$ in factored form?

13. The GCF of $7n^2$ and 21 is ____. 
14. Use the Distributive Property to factor the expression. 
   $$7n^2 + 21 = ____ (____ + ____)

Problem 3  Factoring $ax^2 + bx + c$ when $|a| \neq 1$

**Got It?** What is the expression $4x^2 + 7x + 3$ in factored form? Check your answers.

15. Complete the diagram below.
   $$4x^2 + 7x + 3$$
   $$\square \cdot \square = 12$$

16. Complete the factor pairs of $ac$. Then circle the pair that sums to 7.
   $$\begin{align*}
   (1, \quad & \quad) \\
   (2, \quad & \quad) \\
   (3, \quad & \quad)
   \end{align*}$$

17. Use your answer to Exercise 16 to complete the diagram below.
   $$4x^2 + 7x + 3$$
   $$\begin{align*}
   &\quad\quad\quad\quad\quad\quad\quad + \\
   &\quad\quad\quad\quad\quad\quad\quad +
   \end{align*}$$

   $$= 4x^2 + (4x) + \square + \square$$
   $$= 4x(\quad + \quad) + 3(\quad + \quad)$$

   The expressions inside the parentheses must be equal.

   Use the Distributive Property to factor out the GCF, the part inside the parentheses.

   $$= (4x + 3)(\quad + \quad)$$

Problem 4  Factoring a Perfect Square Trinomial

**Got It?** What is $64x^2 - 16x + 1$ in factored form?

18. Circle the form your answer will have. 
   $$____ (____ + ____)$$

   $$____ (____ - ____)$$
19. Use the justifications to complete each step.

\[ 64x^2 - 16x + 1 \]
\[ \left( \_ \_x \_ \_ \right)^2 - 16x + \left( \_ \_ \_ \_ \right)^2 \]
\[ \left( \_ \_x \_ \_ \right)^2 - 2\left( \_ \_ \_ \_ \_ \_ \_ \right)x + \left( \_ \_ \_ \_ \_ \_ \_ \_ \_ \right)^2 \]

Write the original expression.
Write the first and third terms as squares.
Write the middle term as \((2ac)x\).

20. Write the expression as the square of a binomial.

Lesson Check • Do you UNDERSTAND?

Reasoning Explain how to rewrite the expression \(a^2 - 2ab + b^2 - 25\) as the product of two trinomial factors. \((Hint: Group the first three terms. What type of expression are they?)\)

21. Complete: The first three terms of the expression are a ___.

- perfect square trinomial
- difference of two squares

22. Factor the first three terms of the expression.

23. Rewrite the original expression using the factored form of the first three terms.

24. Complete: The expression you wrote in Exercise 23 is a ___.

- perfect square trinomial
- difference of two squares

25. Circle the expression written as the product of two trinomial factors.

\[ a^2 - 2ab + b^2 \]
\[ (a - b)^2 - 25 \]
\[ (a - b)(-25) \]
\[ (a - b - 5)(a - b + 5) \]

Math Success

Check off the vocabulary words that you understand.

- factor of an expression
- perfect square trinomial
- difference of two squares

Rate how well you can factor quadratic expressions.

<table>
<thead>
<tr>
<th>Need to review</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>Now I get it!</th>
</tr>
</thead>
</table>