Complex Numbers

Vocabulary

Review

1. Circle the square root that is not a real number.

\[ \sqrt{64} \quad \sqrt{6 - (2)(4)} \quad \sqrt{4 - (2)(-6)} \quad \sqrt{(-5)^2} \]

Vocabulary Builder

**conjugate** (adjective)  
KAHN juh gut

**Related Words:** complex numbers, pairs, roots, imaginary solutions

**Math Usage:** The conjugate of the complex number \( a + bi \) is \( a - bi \).

**Main Idea:** Complex solutions occur in conjugate pairs of the form \( a + bi \) and \( a - bi \). The product of complex conjugates is always a real number. You can use complex conjugates to simplify division of complex numbers.

Use Your Vocabulary

Write C if the number pairs are complex conjugate or N if they are not.

2. \( 4 + 3i, 4 - 3i \)  \( \boxed{C} \)

3. \( 5 + \sqrt{2}, 5 - \sqrt{2} \)  \( \boxed{N} \)

4. \( \sqrt{5} - \sqrt{3}i, \sqrt{5} + \sqrt{3}i \)  \( \boxed{C} \)

5. \( 3 + \sqrt{5}i, 3 + \sqrt{-5}i \)  \( \boxed{N} \)

Key Concept  
Square Root of a Negative Real Number

The imaginary unit \( i \) is the complex number whose square is \(-1\). So, \( i^2 = -1 \), and \( i = \sqrt{-1} \).

For any positive real number \( a \), \( \sqrt{-a} = \sqrt{-1} \cdot a = \sqrt{-1} \cdot \sqrt{a} = i\sqrt{a} \).

**Example:** \( \sqrt{-5} = i\sqrt{5} \)

Note that \((\sqrt{-5})^2 = (i\sqrt{5})^2 = i^2(\sqrt{5})^2 = -1 \cdot 5 = -5 \) (not 5).
6. Use $\sqrt{-1} = i$ to complete each equation.

$$\sqrt{-2} = \_\sqrt{2} \quad \sqrt{-3} = i\sqrt{3} \quad \sqrt{-6} = \_ \quad \sqrt{-8} = i\sqrt{8}$$

**Problem 1** Simplifying a Number Using $i$

**Got It?** How do you write the number $\sqrt{-12}$ using the imaginary unit $i$?

7. Circle the expression that is equivalent to $\sqrt{-12}$.

$$\sqrt{-1} \cdot 4(-3) \quad 4\sqrt{-1} \cdot \sqrt{3} \quad \sqrt{-1} \cdot 4 \cdot 3 \quad -2\sqrt{3}$$

8. Simplify the expression you circled in Exercise 7.

9. Using the imaginary unit $i$, $\sqrt{-12} = \_$.  

$$2\sqrt{3}i \quad 4\sqrt{i} \quad 6i \quad 4\sqrt{3}i$$

**Problem 2** Graphing in the Complex Number Plane

**Got It?** What are the graph and absolute value of $5 - i$?

10. Underline the correct words to complete the sentence.

The graph of $5 - i$ is 5 units left/right and 1 unit up/down.

11. Graph the point.

12. Find the absolute value.

$$|5 - i| = \sqrt{(\_ \_ \_ \_)^2 + (\_ \_ \_ \_)^2} \quad \text{Use the Distance Formula.}$$

$$= \sqrt{\_ \_ \_ \_} \quad \text{Simplify powers.}$$

$$= \_ \_ \_ \_ \quad \text{Add.}$$

**Problem 3** Adding and Subtracting Complex Numbers

**Got It?** What is the sum $(7 - 2i) + (-3 + i)$?
13. The sum is found below. Write the justification for each step.

\[(7 - 2i) + (-3 + i)\]  
Write the original expression.

\[7 + (-2i - 3) + i\]  
Property

\[7 + (-3 - 2i) + i\]  
Property

\[(7 - 3) + (-2i + i)\]  
Property

\[4 - i\]  

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**Problem 4**  
**Multiplying Complex Numbers**

*Got It?* What is the product \((7i)(3i)\)?

14. Complete the solution. Justifications are given.

\[(7i)(3i)\]  
Write the original expression.

\[i^2\]  
Multiply.

\[\left(\boxed{\phantom{0}}\right)\left(\boxed{\phantom{0}}\right)\]  
Substitute \(-1\) for \(i^2\).

\[\boxed{\phantom{0}}\]  
Simplify.

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**Problem 5**  
**Dividing Complex Numbers**

*Got It?* What is the quotient \(\frac{5 - 2i}{3 + 4i}\)?

15. Circle the first step in simplifying the fraction.

\[
\text{Find the complex conjugate of } 5 - 2i. \quad \text{Find the complex conjugate of } 3 + 4i.
\]

\[
\text{Find the absolute value of } 5 - 2i. \quad \text{Find the absolute value of } 3 + 4i.
\]

16. Cross out the expression that is NOT equivalent to the quotient.

\[
\frac{15 - 20i - 6i + 8i^2}{9 - 12i + 12i - 16i^2} \quad \frac{15 - 26i + 8i^2}{25} \quad \frac{25 + 10i - 10i - 16i^2}{9 - 12i + 12i - 16i^2}
\]

17. Simplify.

\[
\boxed{\phantom{0}}
\]

18. \(\frac{5 - 2i}{3 + 4i} = \boxed{\phantom{0}}\)
Lesson Check  •  Do you UNDERSTAND?

Error Analysis  Describe and correct the error made in simplifying the expression \((4 - 7i)(4 + 7i)\).

21. Simplify the expression.

\[
(4 - 7i)(4 + 7i) = 16 + 28i - 28i + 49i^2
\]
\[
= 16 - 49
\]
\[
= -33
\]

22. Explain the error shown above.

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Math Success

Check off the vocabulary words that you understand.

☐ imaginary number  ☐ complex number  ☐ complex conjugates

Rate how well you can find complex-number solutions to quadratic equations.