5-2  Polynomials, Linear Factors, and Zeros

**Vocabulary Review**

1. Cross out the expression that does NOT have \( x^4 \) as a factor.

   - \( x^5 \)
   - \( 2x^3 \)
   - \( x^3y \)
   - \( (3x^2)^2 \)

2. Circle the factor tree that shows the prime factorization of \( 14x^2 \).

   ![Factor Tree]

**Vocabulary Builder**

**turning point**  (noun) *TUR ning poynt*

**Related Words:** relative maximum, relative minimum

**Math Usage:** A *turning point* is where the graph of a function changes from going up to going down, or from going down to going up.

**Use Your Vocabulary**

3. Write the letters of the points that describe each *turning point* and intercept.

   - relative maximum
   - relative minimum
   - \( x \)-intercept
   - \( y \)-intercept

4. Place a ✓ if the sentence shows a correct use of the word *turning point*. Place an ✗ if it does not.

   - The turning point in the story was when the hero chose the book instead of the sword.
   - The function has a turning point at \( x = 17 \).
   - A function has a turning point when it crosses the \( x \)-axis.
Problem 1  Writing a Polynomial in Factored Form

Got It? What is the factored form of \( x^3 - x^2 - 12x \)?

5. Factor an \( x \) from each term.
\[
x^3 - x^2 - 12x = x(x^2 - x - 12)
\]

6. Complete the factor table. Then circle the pair of factors that have a sum equal to \(-1\).

7. Complete the factorization using the factors you circled in Exercise 6. Check your answer using FOIL.
\[
x^3 - x^2 - 12x = \boxed{ } \boxed{ } - \boxed{ } \boxed{ }
\]

Problem 2  Finding Zeros of a Polynomial Function

Got It? What are the zeros of \( y = x(x - 3)(x + 5) \)? Graph the function.

8. Use the Zero-Product Property to find the value of \( x \) in each factor.

\( x \)

\( (x - 3) \)

\( (x + 5) \)

9. The zeros of \( y = x(x - 3)(x + 5) \) are \( \boxed{ } \), \( \boxed{ } \), and \( \boxed{ } \).

10. The graph of \( y = x(x - 3)(x + 5) \) crosses the \( x \)-axis at \( \boxed{ } \), \( \boxed{ } \), \( \boxed{ } \), and \( \boxed{ } \).

11. Now graph the function.

Theorem  Factor Theorem

The expression \( x - a \) is a factor of a polynomial if and only if the value \( a \) is a zero of the related polynomial function.

12. Circle the zeros of the polynomial function \( y = (x - 2)(x + 3)(x + 4) \).
13. A polynomial equation \( Q(x) = 0 \) has a solution of \(-2\). Cross out the statement that is NOT true.

One root of the equation is \(-2\).
There is an \(x\)-intercept on the graph of the equation at \(-2\).
A factor of the polynomial is \(x - 2\).

Problem 3 Writing a Polynomial Function From Its Zeros

Got It? What is a quadratic polynomial function with zeros 3 and \(-3\)?

14. The polynomial is found below. Use one of the reasons in the blue box to justify each step.
\[
P(x) = (x - 3)(x + 3)
= x^2 - 3x + 3x - 9
= x^2 - 9
\]

Problem 4 Finding the Multiplicity of a Zero

Got It? What are the zeros of \( f(x) = x^3 - 4x^2 + 4x \)? What are their multiplicities? How does the graph behave at these zeros?

15. Factor \( f(x) = x^3 - 4x^2 + 4x \).

16. The factor \(x\) appears 0 / 1 / 2 times, so the number 0 is a zero of multiplicity __._.

17. The factor \((x - 2)\) appears 0 / 1 / 2 times, so the number 2 is a zero of multiplicity __._.

18. The graph looks close to linear / quadratic at 0 and close to linear / quadratic at 2.

Problem 5 Using a Polynomial Function to Maximize Volume

Got It? Technology The design of a mini digital box camera maximizes the volume while keeping the sum of the dimensions at most 4 inches. If the length must be 1.5 times the height, what is the maximum volume?

19. Complete the reasoning model below to find the volume.

\[
V = \text{the } \underline{\text{volume of the camera}} \cdot \text{the } \underline{\text{length of the camera}} \cdot \text{the } \underline{\text{height of the camera}}.
\]

Given:
- Let \(x = \text{the height of the camera}\).
- \(x = \text{the height of the camera}\).
- \((4 - (x + 1.5)) = \text{the height of the camera}\).

Calculate: [Detailed calculation process involving polynomial functions and optimization techniques]
20. Each graphing calculator screen shows the standard viewing window. Circle the graph of the function.

![Graphs of standard viewing window](image)

21. Use the *maximum* feature on your calculator. The *maximum* volume is _____ in.\(^3\) for a height of _____ in.

22. Now find the length and the width.

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<thead>
<tr>
<th>Length</th>
<th>Width</th>
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23. The dimensions should be approximately _____ in. high by _____ in. long by _____ in. wide.

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**Lesson Check  •  Do you UNDERSTAND?**

**Vocabulary** Write a polynomial function \(h\) that has 3 and -5 as zeros of multiplicity 2.

24. The expressions \((x - 3)\) and \((x + 5)\) are / are not factors of the polynomial \(h\).

25. Write the polynomial in factored form.

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**Math Success**

Check off the vocabulary words that you understand.

- zero
- multiplicity
- relative maximum
- relative minimum

Rate how well you can *find zeros of a polynomial function*.

<table>
<thead>
<tr>
<th>Need to review</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>Now I get it!</th>
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