Bellwork

Which angles are coterminal to $\frac{\pi}{3}$?

- $\frac{7\pi}{3}$
- $\frac{-5\pi}{6}$
- $\frac{4\pi}{3}$
- $\frac{-\pi}{3}$

Find a complementary and supplementary angle.

C: $\frac{\pi}{2} - \frac{4\pi}{5}$

S: $\frac{\pi}{2} - \frac{13\pi}{5}$

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17a) I  b) III  18a) III  b) III  19) IV  b) IV
23)
24)
27) $\frac{13\pi}{6}$, $-\frac{11\pi}{6}$
28) $\frac{19\pi}{6}$, $-\frac{5\pi}{6}$  b) $\frac{\pi}{6}$, $-\frac{23\pi}{6}$
31a) $\frac{\pi}{6}$, $\frac{2\pi}{3}$  b) $\frac{\pi}{4}$, $\frac{3\pi}{4}$
33a) 0.57, 2.14  b) no complement, 1.14
41) II  b) IV
47)
48)
50) 480°, −240°  b) −60°, 300°
54) 44°, 134°  b) no complement, 87°
4.1 Day 2: Radian and degree measure day 2

GOAL: convert between radians and degrees. Find arc length and area of a sector of a circle.

Converting between radians and degrees

$180^\circ = \pi$ radians

Radians to degrees: $\text{Multiply by } \left( \frac{180^\circ}{\pi} \right)$

Degrees to radians: $\text{Multiply by } \left( \frac{\pi}{180^\circ} \right)$

**Example 1:** Convert angle measure from degrees to radians

$135^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{135\pi}{180} = \frac{3\pi}{4}$

$60^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{60\pi}{180} = \frac{\pi}{3}$
**Example 2:** Convert angle measure from radians to degrees

\[
\frac{5\pi}{3} \left( \frac{180^\circ}{\pi} \right) = \frac{3\pi}{4} \left( \frac{180^\circ}{\pi} \right) = 135^\circ
\]

\[
\frac{5 \times 180^\circ}{3} = 300^\circ
\]

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**Arc Length**

**Arc Length:** for a circle of radius \( r \), a central \( \theta \) intercepts an arc of length \( s \) given by

\[
S = r\theta
\]

\( \theta \) is measured in radians

- if \( r = 1 \), then \( s = \theta \) (central angle)
Area of a Sector of a Circle

Area of a Sector of a Circle: For a circle of radius $r$, the area $A$ of a sector of the circle with central angle $\theta$ is given by

$$A = \frac{1}{2} \theta r^2$$

$\theta$ is measured in radians

Example 3: Find the length of the arc on a circle of radius $r$ intercepted by a central angle $\theta$.

A radius of 9 units and central angle of $\frac{\pi}{3} = \theta$

$$S = r \theta$$

$$S = 9 \left( \frac{\pi}{3} \right) = 3\pi = 9.425 \text{ units}$$

A radius of 4 units and central angle of $240^\circ = \theta$

$$r = 4$$

$$\theta = 240^\circ \left( \frac{\pi}{180^\circ} \right) = \frac{4\pi}{3}$$

$$S = r \theta$$

$$S = 4 \left( \frac{4\pi}{3} \right) = \frac{16\pi}{3} \approx 16.755 \text{ units}$$
**Example 4:** Find the radian measure of the central angle of a circle of radius \( r \) that intercepts an arc of length \( s \). \( \Theta = ? \)

A radius of 4 units and arc length of 18

\[
\frac{s}{r} = \Theta
\]

\[
\frac{18}{4} = \Theta \quad \text{\( \Theta = \frac{18}{4} = 4.5 \text{ radians} \)}
\]

\( r = 8 \text{ inches and } s = 22 \text{ inches} \)

\( 2.75 \text{ radians} \)

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**Example 5:** Find the area of the sector of the circle with radius \( r \) and central angle \( \theta \)

A radius of 70 ft. and central angle \( \frac{2\pi}{3} \)

\[
A = \frac{1}{2} \theta r^2 = \frac{1}{2} \left( \frac{2\pi}{3} \right) (10)^2 = \frac{11}{3} (4900) = \frac{49000\pi}{3} = 5131 \text{ ft}^2
\]

A radius of 2.5 feet and central angle of 225° \( \frac{5\pi}{4} \)

\[
225\left( \frac{\pi}{180} \right) = \frac{5\pi}{4} = \Theta
\]

\[
A = \frac{1}{2} \left( \frac{5\pi}{4} \right) (2.5)^2 = 12.7 \text{ ft}^2
\]
Degree measurements with decimals are represented using degrees, minutes, and seconds.

1 degree = __________ minutes  
1 minute = __________ degrees 
1 second = __________ degrees

**Example 6:** convert $85^\circ 18'30''$ to decimal degree

\[
18 \left( \frac{1}{60} \right) = \frac{18}{60} = 0.3 \\
30 \left( \frac{1}{3600} \right) = \frac{30}{3600} = 0.00833 \\
85 + 0.3 + 0.00833 = 85.30833^\circ
\]

**Example 7:** convert the latitude and longitude of Omaha, NE to decimal degrees:

$41^\circ 15'N, 95^\circ 56'W$

\[
41 \left( \frac{1}{60} \right) = 0.25 \\
41 + 0.25 = 41.25^\circ \\
95 \left( \frac{1}{60} \right) = 0.93 \\
95.93^\circ
\]
Example 8: convert the decimal degree to degrees, minutes, seconds. Check using calculator: 125.768°

Example 9: Finding distance on the Earth
Assume that the radius of Earth is 4000 miles (and the Earth is a sphere). Find the distance between San Francisco, CA with latitude 37°47′36″ and Seattle, WA with latitude 47°37′18″.
4.1 day 2 assignment (#2)

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