Bellwork

Solve the triangle for all sides and angles. (hint: use SOH-CAH-TOA)

Angles:
A = \tan^{-1} \left( \frac{4}{10} \right) = 21.8^\circ
B = \tan^{-1} \left( \frac{10}{4} \right) = 68.2^\circ

Sides:
\[ c = \sqrt{10^2 + (-4)^2} = 10.8 \]
\[ \sin 68.2^\circ = \frac{10}{c} \]

In the (x, y) coordinate plane, what is the x-intercept of the line 2x – 5y = 10?
A) -5  B) -2  C) 5  D) 8  E) 10

Chapter 6: Law of Sines and Cosines

Solving non-right triangles
6.1: Law of Sines

GOAL: Solve for missing sides and angles in non-right triangles.

Law of Sines: Non-right triangles (oblique) ASA, AAS, SSA

If ABC is a triangle with sides a, b, and c, then

\[
\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}
\]

*Law of Sines works quickly when you are given two angles and a side (AAS or ASA)*
Example 1: Solve ΔLMN. Round side lengths to the nearest tenth and angle measures to the nearest degree.

\[180° - 112° - 29° = 39°\]

\[
\frac{\sin L}{\sin 39°} = \frac{\sin M}{\sin 112°} \Rightarrow \frac{\sin 29°}{22} = \frac{\sin 29°}{m}
\]

\[m = \frac{22 \sin 29°}{\sin 29°} = 42.1\]

\[n = \frac{22 \sin 39°}{\sin 29°} = 28.6\]

Example 2: Use the given information to solve the triangle.

\[C = 102.3°, B = 28.7° \text{ and } b = 27.4\]

\[
\frac{\sin 28.7°}{27.4} = \frac{\sin 102.3°}{c}
\]

\[
c = \frac{27.4 \sin 28.7°}{\sin 102.3°} = 55.7
\]

\[
\frac{\sin 28.7°}{27.4} = \frac{\sin 49°}{a}
\]

\[a = 43.1\]
*The Ambiguous Case: Law of Sines is a little trickier when given two sides and an angle opposite one of them. (SSA)*

\[ h = b \sin A \] and \( a \) is the same letter as the given angle

Example 3: Find all solutions for the given triangle, if possible. If no solution exists, write no solution. Round side lengths to the nearest tenth and angle measures to the nearest degree.

\[ A = 42^\circ, a = 22, b = 12 \]

\[ h = b \sin A = 12 \sin 42^\circ = 8 \]

\[ 8 < 12 < 22 \]

\[ h < b < a \]

\[ C = 180^\circ - 42^\circ - 21.4^\circ = 116.6^\circ \]

\[ \sin \frac{42^\circ}{22} = \sin \frac{116.6^\circ}{2} \]

\[ C = 29.4^\circ \]
Example 3: Find all solutions for the given triangle, if possible. If no solution exists, write no solution. Round side lengths to the nearest tenth and angle measures to the nearest degree.

\[ C = 85^\circ, c = 15, b = 25 \]

\[
h = 25 \cdot \sin 85^\circ = 24.9
\]

\[ a < h < b \]

**No Solution**

Example 3: Find all solutions for the given triangle, if possible. If no solution exists, write no solution. Round side lengths to the nearest tenth and angle measures to the nearest degree.

\[ A = 105^\circ, a = 73, b = 55 \]

\[ a > b \]

**One Solution**

\[ B = 46.7^\circ \]

\[ C = 28.3^\circ \]

\[ c = 35.8 \]
Example 3: Find all solutions for the given triangle, if possible. If no solution exists, write no solution. Round side lengths to the nearest tenth and angle measures to the nearest degree.

\[ A = 20^\circ 30', a = 12, b = 31 \]

\[
\begin{align*}
A &= 20.5^\circ \\
h &= 31 \sin 20.5^\circ = 10.9 \\
B &= \sin^{-1}\left(\frac{31 \sin 20.5^\circ}{12}\right) = 64.8^\circ \\
C_1 &= 180^\circ - 20.5^\circ - 64.8^\circ = 94.7^\circ \\
C_1 &= \frac{34.2}{12} \\
B_A &= 180^\circ - 64.8^\circ = 115.2^\circ \\
C_2 &= 180^\circ - 20.5^\circ - 115.2^\circ = 44.3^\circ \\
C_2 &= 23.9
\end{align*}
\]

6.1 assignment

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