3.4 Solving logarithm/exponential equations

Bellwork:

Solve by writing in exponential/logarithm form

\[ \log_x 27 = 3 \]
\[ x^3 = 27 \rightarrow x = 3 \]

Use properties of natural log to simplify

\[ \ln e = 1 \]
\[ \ln e^2 = 2 \]

\[ 10^2 = 1000 \]

GOAL:

Solve equations with exponents + logs

Goal 1: Solving logarithm equations

Example 1: Using one-to-one property of logarithms

*Use when...

\[ \log_b x = \log_b y \Rightarrow x = y \]

\[ \log_4 6x = \log_4 12 \]
\[ 6x = 12 \]
\[ x = 2 \]

\[ 2 \log_5 x - \log_5 4 = \log_5 16 \]
\[ 2 \log_5 x = \log_5 16 \]
\[ \log_5 \left( x^2 \right) = \log_5 16 \]
\[ x^2 = 16 \]
\[ x = \pm 4 \]

*Only positive solution

\[ \log_5 (3x + 1) = 2 \]
\[ 5^2 = 3x + 1 \]
\[ 25 = 3x + 1 \]
\[ 2x = 24 \]
\[ x = 8 \]

Example 2: If the variable is in a logarithm change to exponential form

*Use when...

\[ \log_5 (x - 1) = \frac{8}{4} \]
\[ \log_5 (x - 1) = 2 \]
\[ 5 \log_5 (x - 1) = 8 \]
\[ \log_5 (x - 1)^5 = 8 \]
\[ (x - 1)^5 = 5^8 \]
\[ x - 1 = 5 \]
\[ x = 6 \]

\[ 6 + 4 \log_3 (x - 1) = 14 \]
\[ -6 \]
\[ 4 \log_3 (x - 1) = 8 \]
\[ \log_3 (x - 1)^4 = \frac{8}{4} \]
\[ 3^4 = x - 1 \]
\[ x = 10 \]

\[ 2 \ln 5x = 8 \]
\[ \ln 5x = 4 \]
\[ \ln 5 + \ln x = 4 \]
\[ \ln x = 4 - \ln 5 \]
\[ e^4 = 5x \]
\[ x = \frac{e^4}{5} \]
\[ x = 10.92 \]

\[ \ln x - \ln 3 = 0 \]
\[ \ln x = \ln 3 \]
\[ x = 3 \]

\[ \ln \frac{x}{3} = 0 \]
\[ e^0 = \frac{x}{3} \]
\[ 1 = \frac{x}{3} \]
\[ x = 3 \]

\[ 10 \ln x - 3 = 117 \]
\[ 10 \ln x = 120 \]
\[ \ln x = 12 \]
\[ e^{12} = x \]

\[ \log_2 (x + 6) + \log_2 x = 4 \]
\[ \log_2 x(x + 6) = 4 \]
\[ x^2 + 6x = 2^4 \]
\[ x^2 + 6x - 16 = 0 \]
\[ (x + 8)(x - 2) = 0 \]
\[ x = -8, 2 \]
**Goal 2: Solving exponential equations**

**Example 1:** get the same base

\[
3^{x+1} = 27
\]

\[
3^{x+1} = 3^3
\]

\[
x + 1 = 3
\]

\[
x = 2
\]

\[
10^{3x-4} = 0.01
\]

\[
10^{3x-4} = 10^{-2}
\]

\[
3x - 4 = -2
\]

\[
x = \frac{2}{3}
\]

\[
9^{2x} = 3^{x-6}
\]

\[
3^{2(2x)} = 3^{x-6}
\]

\[
4x = x - 6
\]

\[
3x = -6
\]

\[
x = -2
\]

\[
\text{X negative, not okay}
\]

\[
4x^2 = 16^x
\]

\[
x^2 = 4(x-3)
\]

\[
x + 2 = 2x - 6
\]

\[
x = -8
\]

**Example 2:** If the variable is in the exponent change to ______________.

\[
3^x = 11
\]

\[
\log_3 11 = x
\]

\[
x = \log_3 11
\]

\[
x = 2.18
\]

\[
\log e^x = \ln 23
\]

\[
x = \ln 23
\]

\[
x = 3.135
\]

\[
10^{x-3} + 4 = 21
\]

\[
10^{x-3} = 17
\]

\[
\log_{10} 17 = x - 3
\]

\[
x = \log_{10} 17 + 3
\]

\[
x = 4.23
\]

\[
5e^{2x} - 10 = -2
\]

\[
5e^{2x} = 8
\]

\[
\ln e^{2x} = \ln 8
\]

\[
2x = \ln 8
\]

\[
x = \frac{\ln 8}{2}
\]

\[
x = 0.285
\]

\[
2(3^{2x-5}) - 4 = 11
\]

\[
\log_3 2^{2x-5} = \log_3 7.5
\]

\[
(2x - 5) \log_3 2 = \log_3 7.5
\]

\[
2x - 5 = 1.834
\]

\[
x = 3.917
\]

\[
10^{4x-2} = 12
\]

\[
(4x - 2) \log 10 = \log 12
\]

\[
4x - 2 = \log 12
\]

\[
x = 0.770
\]

**Summary:** Pick the example you thought was most difficult...what did you learn from it?
251 # 17, 19, 21, 24, 29, 31, 37, 38, 43, 44, 49, 51, 59, 81, 88, 90, 93, 108